Preprint typeset in JHEP style - HYPER VERSION

Feynman Rules for the Rational Part of the Standard Model One-loop Amplitudes in the 't Hooft-Veltman γ_5 Scheme

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Abstract: We study Feynman rules for the rational part R of the Standard Model amplitudes at one-loop level in the 't Hooft-Veltman γ_5 scheme. Comparing our results for quantum chromodynamics and electroweak 1-loop amplitudes with that obtained based on the Kreimer-Korner-Schilcher (KKS) γ_5 scheme, we find the latter result can be recovered when our γ_5 scheme becomes identical (by setting $g5s = 1$ in our expressions) with the KKS scheme. As an independent check, we also calculate Feynman rules obtained in the KKS scheme, finding our results in complete agreement with formulae presented in the literature. Our results, which are studied in two different γ_5 schemes, may be useful for clarifying the γ_5 problem in dimensional regularization. They are helpful to eliminate or find ambiguities arising from different dimensional regularization schemes.

Keywords: NLO, Electroweak interactions, Quantum chromodynamics.

Contents

1. Introduction

The operation of the Large Hadron Collider (LHC) at CERN has opened a new era in high energy physics. There are many important topics for LHC physics, particularly in hunting Higgs boson(s) and searching for new physics signals beyond the standard model (BSM). However, the existence of large standard model (SM) background makes such studies difficult. In this regard, the next-to-leading order (NLO) corrections to the standard model calculations are necessary and helpful, because these corrections may provide a much improved scale dependence and more reliable estimations in separating SM background from new physics signals. Since the Les Houches workshop in 2005, many NLO corrections in processes with four or more final state particles at the LHC have been successfully performed. They are, e.g., $pp \to t\bar{t}b\bar{b}+X$ [1, 2], $pp \to V+3jets$ [3], $pp \to W^{\pm}+4jets$ [4], $pp \to VV+2jets$ [5, 6], $pp \to t\bar{t}+2jets$ [7], $pp(\bar{p}) \to W^+W^-b\bar{b}$ [8], $pp \to b\bar{b}b\bar{b}+X$ [9] and $pp \to W\gamma\gamma+jet$[10]. However, a few SM processes of similar complexity, which are relevant to search for Higgs boson(s) or detection of new particles in physics BSM, are still not available at NLO level [11], such as $pp \to t\bar{t}t\bar{t}$.

Recently, automatic one-loop calculations have become a feasible task after several new and efficient algorithms have been proposed. Some of the most notable methods are the Unitarity [12] based techniques such as the Ossola, Papadopoulos, and Pittau (OPP) reduction method [13], which reduces the computation of one-loop amplitudes to a problem with a complexity similar to a tree level calculation. Most of the aforementioned processes are computed within this method or its variations. Such approaches rely on the master integrals of one-loop amplitudes that can be simply represented as a linear combination of up to 4-point scalar integrals, i.e., those of boxes, triangles, bubbles and tadpoles. A one-loop amplitude $\mathcal{A}$ can be written as

$$\mathcal{A} = \sum_i d_i \; Box_i + \sum_i c_i \; Triangle_i + \sum_i b_i \; Bubble_i + \sum_i a_i \; Tadpole_i + R.$$

In the OPP framework, the coefficients d_i, c_i, b_i, a_i and R_1 ($R = R_1 + R_2$) can be obtained from the cut constructible (CC) part of the amplitude, where the numerators of loop integrals are dealt with in 4 dimensions, and R_2 should be derived separately [14]. With the ultraviolet nature of R_2 [1, 15], we can establish Feynman rules for this part. Here we directly call R_2 the rational term R for brevity. In fact, this approach is more efficient than applying the Unitarity reduction method in $d = 4 - 2\epsilon$ dimensions in practical calculations [11]. Algebraically, even in other methods, such as the conventional Passarino-Veltman reduction procedure [16], calculations of R are also useful in multi-leg processes [1].

Several previous works studied calculations of the Feynman rules for R, and a set of Feynman rules for R in the Standard Model (SM) under one γ_5 strategy in the 't Hooft-Feynman gauge, R_ξ gauge, and Unitary gauge has been reported [17, 18, 19], and a package *R2SM* written in FORM is also available [20]. Moreover, some simplifications in extracting rational terms were suggested recently in Ref.[6]. The primary aim of the present paper is to give our result in another γ_5 regularization scheme, i.e. the 't Hooft -Veltman γ_5 scheme. In addition, we would also like to check the results presented in Refs.[17, 18].

This paper is organized as follows. In Section 2, we recall the theory for the rational part R, and set our regularization schemes and γ_5 strategies. The notations and Feynman rules are given in Section 3. Our results in two γ_5 schemes are expressed in Section 4. Finally, a summary is given in Section 5.

2. The theory of R and regularization schemes

In this section, we will describe the origin of the rational part R, and discuss the dimensional regularization scheme (DREG) in two γ_5 strategies. Due to the divergences in a one-loop calculation, we compute our one-loop integrals in $d = 4 - 2\epsilon$

dimensions to regularize the divergences that appear in 4 dimensions. A generic N-point one-loop (sub-) amplitude reads

$$\mathcal{A}_N = \int d^d\bar{q}\frac{\bar{N}(\bar{q})}{\bar{D}_1\bar{D}_2...\bar{D}_N}, \bar{D}_k = (\bar{q}+p_k)^2 - (m_k)^2, \quad \bar{q} = q + \tilde{q}, \tag{2.1}$$

where the bar means quantities defined in d dimensions, while the tilde in $d-4$ dimensions. All quantities of the external particles such as external polarization vectors and external momenta p_i are maintained in 4 dimensions.

In numerator the function $\bar{N}(\bar{q})$ can be split into a 4-dimensional part $N(q)$ and a $(d-4)$-dimensional part $\tilde{N}(\tilde{q}^2, \epsilon, q)$, i.e.,

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, \epsilon, q). \tag{2.2}$$

Here the $\tilde{N}(\tilde{q}^2, \epsilon, q)$ part is related to R:

$$R \equiv \int d^d\bar{q}\frac{\tilde{N}(\tilde{q}^2, \epsilon, q)}{\bar{D}_1\bar{D}_2...\bar{D}_N}. \tag{2.3}$$

Feynman rules for the R effective vertices can be obtained from all possible one-particle irreducible Green functions, which are enough up to 4 external legs in the SM, accounting for the ultraviolet nature of the rational terms.

The d-dimensional momentum $\bar{q}$, the d-dimensional metric tensor $\bar{g}_{\bar{\mu}\bar{\nu}}$, and the d-dimensional Dirac matrices $\bar{\gamma}_{\bar{\mu}}$ are all split into a 4-dimensional part and a -2ϵ-dimensional part

$$\bar{q}_{\bar{\mu}} = q_\mu + \tilde{q}_{\tilde{\mu}}, \;\; \bar{g}_{\bar{\mu}\bar{\nu}} = g_{\mu\nu} + \tilde{g}_{\tilde{\mu}\tilde{\nu}}, \;\; \bar{\gamma}_{\bar{\mu}} = \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}}. \tag{2.4}$$

Instead of performing supersymmetry-preserving but technically complicated dimensional reduction (DRED) [21], we use DREG for convenience. Therefore, the d-dimensional metric tensor $\bar{g}_{\mu\nu}$, the -2ϵ-dimensional tensor $\tilde{g}_{\mu\nu}$, and the 4-dimensional tensor $g_{\mu\nu}$ satisfy following relations :

$$\begin{gathered}\bar{g}_{\mu\rho}g^\rho_\nu = g_{\mu\nu}, \;\; \bar{g}_{\mu\rho}\tilde{g}^\rho_\nu = \tilde{g}_{\mu\nu}, \;\; \tilde{g}_{\mu\rho}g^\rho_\nu = 0, \;\; \bar{g}_{\mu\rho}\bar{g}^\rho_\nu = \bar{g}_{\mu\nu}, \\ g_{\mu\rho}g^\rho_\nu = g_{\mu\nu}, \;\; \tilde{g}_{\mu\rho}\tilde{g}^\rho_\nu = \tilde{g}_{\mu\nu}, \;\; \bar{g}^\mu_\mu = d, \;\; g^\mu_\mu = 4, \;\; \tilde{g}^\mu_\mu = -2\epsilon.\end{gathered} \tag{2.5}$$

In our regularization we set $d > 4$, so that

$$q^2 \to q^2 + \tilde{q}^2. \tag{2.6}$$

To maintain the advantages of the helicity method for loop calculations, we choose the four dimensional helicity (FDH) [22] and 't Hooft-Veltman (HV) schemes [23] in the present paper

$$\begin{aligned} R\Big|_{HV} &= \int d^d\bar{q}\frac{\tilde{N}(\tilde{q}^2, \epsilon, q)}{\bar{D}_1\bar{D}_2...\bar{D}_N}, \\ R\Big|_{FDH} &= \int d^d\bar{q}\frac{\tilde{N}(\tilde{q}^2, \epsilon = 0, q)}{\bar{D}_1\bar{D}_2...\bar{D}_N}. \end{aligned} \tag{2.7}$$

The last comment in this section refers to our γ_5 strategies. We choose two schemes here. One scheme is identical to that used in Refs.[17, 18], which was proposed by Kreimer, Kȯrner, and Schilcher [24] and we call it the KKS scheme for brevity, and the other one is the 't Hoof-Veltman scheme [23, 25, 26], which gives new results in the present paper. These two schemes are algebraically consistent, at least at one-loop level. In the KKS scheme, we define a unique generator which anti-commutates with all other generators in Clifford algebra, as γ_5. So the anti-commutation relation $\{\gamma_5, \bar{\gamma}_{\bar{\mu}}\} = 0$ is maintained. To obtain the anomaly, we give up the cyclic relation in the ordinary Dirac trace, and use a projection relation onto four dimensional subspace

$$Tr(\gamma_5 \bar{\gamma}_{\bar{\mu}_1} ... \bar{\gamma}_{\bar{\mu}_{2k}}) \equiv tr(\mathcal{P}(\gamma_5) \bar{\gamma}_{\bar{\mu}_1} ... \bar{\gamma}_{\bar{\mu}_{2k}}), \tag{2.8}$$

where $\mathcal{P}(\gamma_5) \equiv \frac{i}{4!}\ \varepsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$ with Lorentz indexes of the totally antisymmetric tensor $\varepsilon_{\mu_1\mu_2\mu_3\mu_4}$ all in 4 dimensions. To perform calculations for some specific processes unambiguously, we choose a unique "special vertex", called the "reading point", in all Feynman diagrams. All γ_5's are anti-commuted to reach the "reading point" before performing the d-dimensional algebra calculation. This treatment generally produces a term that is proportional to the total antisymmetric ε tensor with different "reading points". The HV scheme defines $\gamma_5 \equiv \frac{i}{4!}\ \varepsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$ in d dimensions. Therefore, the anti-commutation relation is violated by

$$\{\gamma_5, \gamma_\mu\} = 0,\ \ [\gamma_5, \tilde{\gamma}_{\tilde{\mu}}] = 0. \tag{2.9}$$

This definition explicitly forbids us to deal with covariant γ algebra and often makes calculations very inconvenient. Nevertheless, it is to date the only known scheme within DREG that has been demonstrated to be consistent in all orders. Actually, the violation of anti-commutation also needs to include an extra finite renormalization by hand in calculations to restore the Ward identities. For instance in the W boson decays to hadronic partons, the Ward identity is spoiled in the HV γ_5 scheme at one loop level. The $\mathcal{O}(\alpha_s)$ corrections should include an axial-vector current renormalization

$$\Gamma^{ren}_{\mu 5} = \left(1 - \frac{\alpha_s\, C_F}{2\pi}(1 + \lambda_{HV})\right)\Gamma^{bare}_{\mu 5}, \tag{2.10}$$

where λ_{HV} is defined in the next section and $C_F = \frac{N_c^2-1}{2N_c}$ is implicitly understood. This non-anticommuting γ_5 scheme was also discussed from the action principles in Ref.[27]. Moreover, some issues about dimensional renormalization with γ_5 can also be found in Refs.[27, 28].

3. Notations and Feynman rules

In this section, we will briefly describe our notations and tree level Feynman rules that will be used in the next section.

Two parameters, λ_{HV} and $g5s$, are introduced in our formulae to denote the different DREG and γ_5 schemes. Here, $\lambda_{HV} = 1(0)$ corresponds to the HV (FDH) regularization scheme, while $g5s = 1(-1)$ corresponds to the KKS (HV) γ_5 scheme. Our notations are as follows: $L_1 = e, L_2 = \mu, L_3 = \tau, L_4 = \nu_e, L_5 = \nu_\mu, L_6 = \nu_\tau, Q_1 = u, Q_2 = d, Q_3 = s, Q_4 = c, Q_5 = b, Q_6 = t$. In addition, $e_1 = e, e_2 = \mu, e_3 = \tau, \nu_1 = \nu_e, \nu_2 = \nu_\mu, \nu_3 = \nu_\tau, U_1 = u, U_2 = c, U_3 = t, D_1 = d, D_2 = s, D_3 = b$. The vector boson fields are denoted by A,Z and $W^\pm$ with the generic symbol V, and the physical scalar Higgs field and the scalar goldstone bosons are written as H, ϕ^0, and $\phi^\pm$, respectively. Fermions are also generically symbolized by F, while the mass, charge, and the third isospin component of F are denoted by m_F, Q_F, and I_{3F}, respectively. c_w and s_w respectively denote sine and cosine of the Weinberg angle. N_c is the number of colors and V_{u_i,d_j} refers to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. $\Omega^\pm \equiv \frac{1\pm\gamma_5}{2}$ are the chiral projector operators. Finally, g_s is the coupling constant of strong interaction, while e is the coupling constant of QED.

As we use FeynArts [29] to generate all Feynman amplitudes, our conventions for tree level Feynman rules follow Ref.[30]. Due to the violation of anti-commutation relation in HV γ_5 scheme, i.e. $\{\gamma_5, \bar{\gamma}_{\bar{\mu}}\} \neq 0$, some modifications to conventional Feynman rules are introduced. For coupling to left-handed fields the symmetrically defined vertex $\frac{1}{2}(1+\gamma_5)\bar{\gamma}_{\bar{\mu}}(1-\gamma_5) = \gamma_\mu(1-\gamma_5)$, instead of $\bar{\gamma}_{\bar{\mu}}(1-\gamma_5)$, should be used [31]. Especially, the tree level Feynman rules of vertices $F\bar{F}Z$ should be modified as follows

$$\frac{ie}{c_w s_w}\left(I_{3F}\bar{\gamma}_{\bar{\mu}}\Omega^- - s_w^2 Q_F \bar{\gamma}_{\bar{\mu}}\right) \;\rightarrow\; \frac{ie}{c_w s_w}\left(I_{3F}\gamma_\mu\Omega^- - s_w^2 Q_F \bar{\gamma}_{\bar{\mu}}\right), \tag{3.1}$$

and similarly to $F_1\bar{F}_2W^\pm$ vertices.

4. Results

In this section, our results in 't Hooft-Feynman gauge are presented. It is easy to check that if one sets $g5s = 1$, the results in [17, 18] are all recovered.

4.1 Effective vertices in QCD

In this subsection, we give a complete list of all non-vanishing R effective vertices in QCD, where all the internal lines in one-particle irreducible diagrams represent QCD particles.

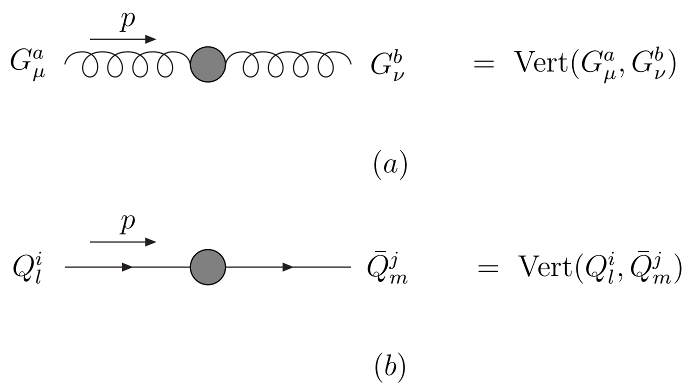


Figure 1: All possible non-vanishing 2-point vertices in QCD.

4.1.1 QCD effective vertices with 2 external legs

All possible non-vanishing 2-point vertices in QCD are shown in Fig.1.

Gluon-Gluon vertex

The corresponding effective vertex is shown in Fig.1 (a) with the following expression

$$\mathrm{Vert}(G^a_\mu, G^b_\nu) = \frac{i g_s^2 N_c}{48\pi^2}\, \delta^{ab} \left[\frac{p^2}{2} g_{\mu\nu} + \lambda_{HV} \left(g_{\mu\nu} p^2 - p_\mu p_\nu \right) + \sum_Q \frac{p^2 - 6 m_Q^2}{N_c} g_{\mu\nu} \right] \tag{4.1}$$

Quark-Quark vertices

The corresponding diagram is shown in Fig.1 (b) with the following expression

$$\mathrm{Vert}(Q^i_l, \bar{Q}^j_m) = \frac{i g_s^2}{16\pi^2}\, \frac{N_c^2 - 1}{2 N_c}\, \delta^{ij}\, \delta_{lm}\, \left(-\not{p} + 2 m_{Q_l} \right) \lambda_{HV}. \tag{4.2}$$

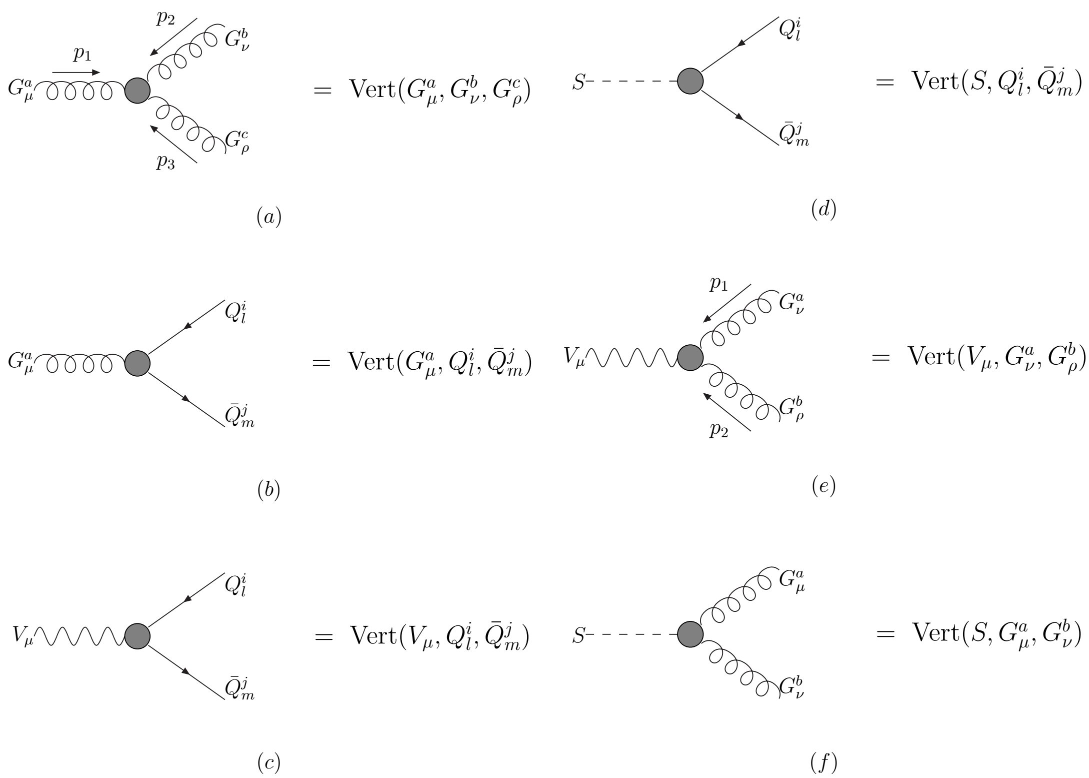


Figure 2: All possible non-vanishing 3-point vertices in QCD.

4.1.2 QCD effective vertices with 3 external legs

All possible non-vanishing 3-point vertices in QCD are shown in Fig.2.

Gluon-Gluon-Gluon vertex

The corresponding diagram is shown in Fig.2 (a) with the following expression for N_f-flavor quarks

$$\mathrm{Vert}(G^a_\mu, G^b_\nu, G^c_\rho) = -\frac{g_s^3 N_c}{48\pi^2}\left(\frac{7}{4} + \lambda_{HV} + \frac{2N_f}{N_c}\right) f^{abc}\, V_{\mu\nu\rho}(p_1, p_2, p_3), \tag{4.3}$$

where

$$V_{\mu\nu\rho}(p_1, p_2, p_3) = g_{\mu\nu}(p_2 - p_1)_\rho + g_{\nu\rho}(p_3 - p_2)_\mu + g_{\rho\mu}(p_1 - p_3)_\nu. \tag{4.4}$$

Gluon-Quark-Quark vertex

The corresponding diagram is shown in Fig.2 (b) with the following expression

$$\mathrm{Vert}(G^a_\mu, Q^i_l, \bar{Q}^j_m) = \delta_{lm}\ \frac{ig_s^3}{16\pi^2}\ T^a_{ji}\ \frac{N_c^2-1}{2N_c}\ \gamma_\mu\,(1+\lambda_{HV})\,. \tag{4.5}$$

Vector-Quark-Quark vertices

A typical Vector-Quark-Quark vertex is shown in Fig.2 (c) with the following expression

$$\begin{aligned}\mathrm{Vert}(V_\mu, Q^i_l, \bar{Q}^j_m) = &-\frac{g_s^2}{16\pi^2}\ \frac{N_c^2-1}{2N_c}\ \delta^{ij}\ \ (1+\lambda_{HV})\\ &\gamma_\mu\left(v^{VQ_l\bar{Q}_m} + g5s\ a^{VQ_l\bar{Q}_m}\gamma_5\right).\end{aligned} \tag{4.6}$$

The actual values of V, Q_l, $\bar{Q}_m$, $v^{VQ_l\bar{Q}_m}$ and $a^{VQ_l\bar{Q}_m}$ are

$$\begin{aligned}
v^{AQ_l\bar{Q}_m} &= -ieQ_{Q_l}\delta_{lm},\\
a^{AQ_l\bar{Q}_m} &= 0,\\
v^{ZQ_l\bar{Q}_m} &= \frac{ies_w}{c_w}\left(\frac{I_{3Q_l}}{2s_w^2} - Q_{Q_l}\right)\delta_{lm},\\
a^{ZQ_l\bar{Q}_m} &= -\frac{ieI_{3Q_l}}{2s_wc_w}\delta_{lm},\\
v^{W^-U_l\bar{D}_m} &= V^\dagger_{D_m,U_l}\frac{ie}{2\sqrt{2}s_w},\\
a^{W^-U_l\bar{D}_m} &= -V^\dagger_{D_m,U_l}\frac{ie}{2\sqrt{2}s_w},\\
v^{W^+D_l\bar{U}_m} &= V_{U_m,D_l}\frac{ie}{2\sqrt{2}s_w},\\
a^{W^+D_l\bar{U}_m} &= -V_{U_m,D_l}\frac{ie}{2\sqrt{2}s_w}.
\end{aligned} \tag{4.7}$$

Scalar-Quark-Quark vertices

The generic diagram is shown in Fig.2 (d) with the following expression

$$\mathrm{Vert}(S, Q_l^i, \bar{Q}_m^j) = -\frac{g_s^2}{8\pi^2}\ \frac{N_c^2-1}{2N_c}\ \delta^{ij}\ \ (1+\lambda_{HV}) \\ \left(v^{SQ_l\bar{Q}_m} + g5s\ a^{SQ_l\bar{Q}_m}\gamma_5\right). \tag{4.8}$$

The actual values of S, Q_l, $\bar{Q}_m$, $v^{SQ_l\bar{Q}_m}$ and $a^{SQ_l\bar{Q}_m}$ are

$$\begin{aligned}
v^{HQ_l\bar{Q}_m} &= -\delta_{lm}\ \frac{i\ e\ m_{Q_l}}{2m_W s_w}, & a^{HQ_l\bar{Q}_m} &= 0, \\
v^{\phi^0 U_l\bar{U}_m} &= 0, & a^{\phi^0 U_l\bar{U}_m} &= -\delta_{lm}\ \frac{e\ m_{U_l}}{2m_W s_w}, \\
v^{\phi^0 D_l\bar{D}_m} &= 0, & a^{\phi^0 D_l\bar{D}_m} &= \delta_{lm}\ \frac{e\ m_{D_l}}{2m_W s_w}, \\
v^{\phi^- U_l\bar{D}_m} &= V^\dagger_{D_m,U_l}\frac{i\ e}{2\sqrt{2}m_W s_w}\left(m_{U_l} - m_{D_m}\right), \\
a^{\phi^- U_l\bar{D}_m} &= V^\dagger_{D_m,U_l}\frac{i\ e}{2\sqrt{2}m_W s_w}\left(m_{U_l} + m_{D_m}\right), \\
v^{\phi^+ D_l\bar{U}_m} &= V_{U_m,D_l}\frac{i\ e}{2\sqrt{2}m_W s_w}\left(m_{U_m} - m_{D_l}\right), \\
a^{\phi^+ D_l\bar{U}_m} &= -V_{U_m,D_l}\frac{i\ e}{2\sqrt{2}m_W s_w}\left(m_{U_m} + m_{D_l}\right).
\end{aligned} \tag{4.9}$$

Vector-Gluon-Gluon vertices

The generic diagram is shown in Fig.2 (e) with the following expression

$$\mathrm{Vert}(V_\mu, G_\nu^a, G_\rho^b) = -\frac{ig_s^2}{12\pi^2}\ \delta^{ab}\ \varepsilon_{\mu\nu\rho(p_1-p_2)}\ \sum_l a^{VQ_l\bar{Q}_l}, \tag{4.10}$$

with all of the expressions of $a^{VQ_l\bar{Q}_l}$ given in Eq.(4.7).

Scalar-Gluon-Gluon vertices

The generic diagram is shown in Fig.2 (f) with the following expression

$$\mathrm{Vert}(S, G_\mu^a, G_\nu^b) = \frac{g_s^2}{8\pi^2}\ \delta^{ab}\ g_{\mu\nu}\ \sum_l \left(v^{SQ_l\bar{Q}_l} m_{Q_l}\right), \tag{4.11}$$

with all of the expressions of $v^{SQ_l\bar{Q}_l}$ given in Eq.(4.9).

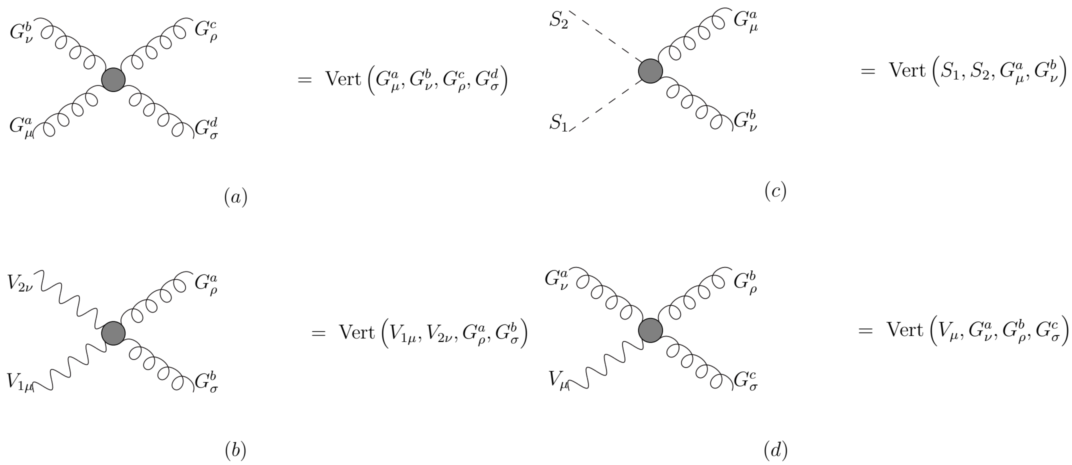


Figure 3: All possible non-vanishing 4-point vertices in QCD.

4.1.3 QCD effective vertices with 4 external legs

All possible non-vanishing 4-point vertices in QCD are shown in Fig.3.

Gluon-Gluon-Gluon-Gluon vertex

The corresponding diagram is shown in Fig.3 (a) with the following expression for N_f-flavor quarks

$$\text{Vert}\left(G^a_\mu, G^b_\nu, G^c_\rho, G^d_\sigma\right) = \frac{ig_s^4}{48\pi^2}\ \left(C_1\ g_{\mu\nu}g_{\rho\sigma} + C_2\ g_{\mu\rho}g_{\nu\sigma} + C_3\ g_{\mu\sigma}g_{\nu\rho}\right), \quad (4.12)$$

where

$$\begin{aligned}
C_1 &= Tr(\{T^a, T^b\}\{T^c, T^d\})\ \left(5N_c + 2\lambda_{HV}N_c + 6N_f\right) \\
&\quad - \left(Tr(T^aT^cT^bT^d) + Tr(T^aT^dT^bT^c)\right)\left(12N_c + 4\lambda_{HV}N_c + 10N_f\right) \\
&\quad - \left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right), \\
C_2 &= C_1(b \leftrightarrow c) = Tr(\{T^a, T^c\}\{T^b, T^d\})\ \left(5N_c + 2\lambda_{HV}N_c + 6N_f\right) \\
&\quad - \left(Tr(T^aT^bT^cT^d) + Tr(T^aT^dT^cT^b)\right)\left(12N_c + 4\lambda_{HV}N_c + 10N_f\right) \\
&\quad - \left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right), \\
C_3 &= C_1(b \leftrightarrow d) = Tr(\{T^a, T^d\}\{T^c, T^b\})\ \left(5N_c + 2\lambda_{HV}N_c + 6N_f\right) \\
&\quad - \left(Tr(T^aT^cT^dT^b) + Tr(T^aT^bT^dT^c)\right)\left(12N_c + 4\lambda_{HV}N_c + 10N_f\right) \\
&\quad - \left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right).
\end{aligned} \quad (4.13)$$

Vector-Vector-Gluon-Gluon vertices

The generic diagram is shown in Fig.3 (b) with the following expression

$$\begin{aligned}\mathrm{Vert}\left(V_{1\mu},V_{2\nu},G^a_\rho,G^b_\sigma\right) = -\frac{ig_s^2}{24\pi^2}\,\delta^{ab}\,\sum_{l,m}\Big[\Big(&v^{V_1Q_l\bar{Q}_m}v^{V_2Q_m\bar{Q}_l}+a^{V_1Q_l\bar{Q}_m}a^{V_2Q_m\bar{Q}_l}\Big)\\ &(g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho})+\frac{1-g5s}{2}\,8a^{V_1Q_l\bar{Q}_m}a^{V_2Q_m\bar{Q}_l}\\ &g_{\mu\nu}g_{\rho\sigma}\Big]\,. \qquad (4.14)\end{aligned}$$

All of the expressions for $v^{VQ_l\bar{Q}_m}$,$a^{VQ_l\bar{Q}_m}$ are given in Eq.(4.7).

Scalar-Scalar-Gluon-Gluon vertices

The generic diagram is shown in Fig.3 (c) with the following expression

$$\begin{aligned}\mathrm{Vert}\left(S_1,S_2,G^a_\mu,G^b_\nu\right) = \frac{ig_s^2}{24\pi^2}\,\delta^{ab}\,\sum_{l,m}\Big[3\,\Big(&v^{S_1Q_l\bar{Q}_m}v^{S_2Q_m\bar{Q}_l}-a^{S_1Q_l\bar{Q}_m}a^{S_2Q_m\bar{Q}_l}\Big)\\ -&\frac{1-g5s}{2}\,8a^{S_1Q_l\bar{Q}_m}a^{S_2Q_m\bar{Q}_l}\Big]\,g_{\mu\nu}. \qquad (4.15)\end{aligned}$$

All of the expressions for $v^{SQ_l\bar{Q}_m}$,$a^{SQ_l\bar{Q}_m}$ are given in Eq.(4.9).

Vector-Gluon-Gluon-Gluon vertices

The generic diagram is shown in Fig.3 (d) with the following expression

$$\begin{aligned}\mathrm{Vert}\left(V_\mu,G^a_\nu,G^b_\rho,G^c_\sigma\right) = -\frac{g_s^3}{24\pi^2}\,\sum_{l}\Big[&Tr(T^a\{T^b,T^c\})\;v^{VQ_l\bar{Q}_l}\,(g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho})\\ &-i9\;Tr(T^a[T^b,T^c])\;a^{VQ_l\bar{Q}_l}\varepsilon_{\mu\nu\rho\sigma}\Big]\,, \qquad (4.16)\end{aligned}$$

with $\{A,B\}\equiv AB+BA$ and $[A,B]\equiv AB-BA$. All of the expressions for $v^{VQ_l\bar{Q}_l}$,$a^{VQ_l\bar{Q}_l}$ are given in Eq.(4.7).

4.2 Effective vertices in Electroweak part

A complete list of the non-vanishing R effective vertices in the Electroweak part is given below.

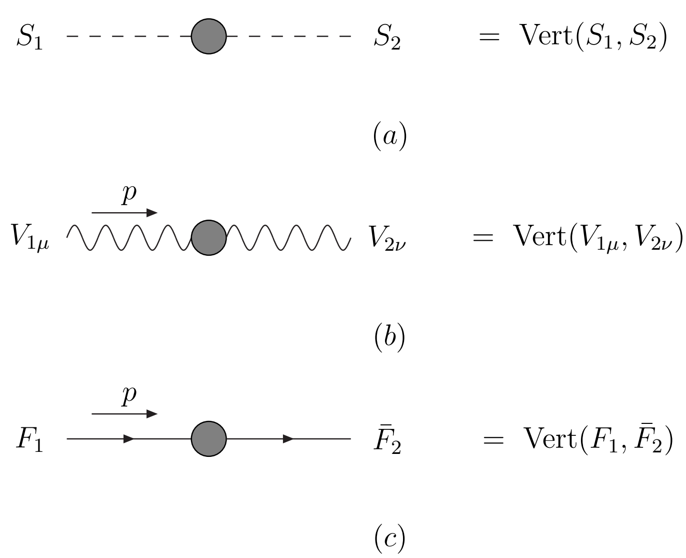


Figure 4: All possible non-vanishing 2-point vertices in EW.

4.2.1 Electroweak effective vertices with 2 external legs

All possible non-vanishing 2-point vertices in Electroweak (EW) are shown in Fig.4.

Scalar-Scalar vertices

The generic effective vertex is shown in Fig.4 (a) with the following expression

$$\text{Vert}\,(S_1, S_2) = \frac{ie^2}{16\pi^2 s_w^2}\, C^{S_1 S_2}, \tag{4.17}$$

with the actual value of S_1, S_2 and $C^{S_1 S_2}$

$$
\begin{aligned}
C^{H\phi^0} &= 0, \\
C^{HH} &= \frac{1-6\lambda_{HV}}{2}\left(1+\frac{1}{2c_w^4}\right)m_W^2 - \left(1+\frac{1}{2c_w^2}\right)\frac{p^2}{12}+K, \\
C^{\phi^0\phi^0} &= \frac{m_W^2}{4}+\frac{m_H^2}{8c_w^2}+\frac{1-4\lambda_{HV}}{4}\left(1+\frac{1}{2c_w^4}\right)m_W^2 \\
&\quad -\left(1+\frac{1}{2c_w^2}\right)\frac{p^2}{12}+g5s\ K, \\
C^{\phi^-\phi^+} &= \frac{m_W^2}{4c_w^2}+\frac{m_H^2}{8}+\frac{\left(3-4\lambda_{HV}\right)c_w^4-2c_w^2+\left(\frac{1}{2}-2\lambda_{HV}\right)}{4c_w^4}m_W^2 \\
&\quad -\left(1+\frac{1}{2c_w^2}\right)\frac{p^2}{12}+\frac{1}{2m_W^2}\left\{\frac{1+g5s}{2}\ \sum_l\left(m_{e_l}^2\left(m_{e_l}^2-\frac{p^2}{3}\right)\right)\right. \\
&\quad +N_c\sum_{l,m}\left[V_{U_l,D_m}V_{D_m,U_l}^{\dagger}\left(\frac{1+g5s}{2}\ \left(m_{U_l}^2+m_{D_m}^2\right)\right.\right. \\
&\quad \left.\left.\left.-\frac{1-g5s}{2}\ 2m_{U_l}m_{D_m}\right)\left(m_{U_l}^2+m_{D_m}^2-\frac{p^2}{3}\right)\right]\right\},
\end{aligned}
\tag{4.18}
$$

where

$$
K=\sum_l\left[\frac{m_{e_l}^2}{m_W^2}\left(m_{e_l}^2-\frac{p^2}{6}\right)\right]+N_c\sum_l\left[\frac{m_{Q_l}^2}{m_W^2}\left(m_{Q_l}^2-\frac{p^2}{6}\right)\right]. \tag{4.19}
$$

To compare our expressions with that in Ref.[18], we use $m_{\phi^0}^2 = m_Z^2 = \frac{m_W^2}{c_w^2}$ and $m_{\phi^\pm}^2 = m_W^2$ in above expressions and fix the masses of neutrinos to be zero.

Vector-Vector vertices

The generic effective vertex is shown in Fig.4 (b) with the following expression

$$
\mathrm{Vert}\left(V_{1\mu},V_{2\nu}\right)=\frac{ie^2}{\pi^2}\ \left(C_1^{V_1V_2}\ p_\mu p_\nu+C_2^{V_1V_2}\ g_{\mu\nu}\right), \tag{4.20}
$$

with the actual values of V_1, V_2, $C_1^{V_1V_2}$ and $C_2^{V_1V_2}$

$$
\begin{aligned}
C_1^{AA} &= -\frac{\lambda_{HV}}{24}, \\
C_2^{AA} &= \frac{1}{8}\left[\frac{(1+2\lambda_{HV})\,p^2}{6} - m_W^2\right] - \frac{1}{4}\left\{\sum_l \left[Q_{L_l}^2\left(m_{L_l}^2 - \frac{p^2}{6}\right)\right]\right. \\
&\quad \left. + N_c \sum_l \left[Q_{Q_l}^2\left(m_{Q_l}^2 - \frac{p^2}{6}\right)\right]\right\}, \\
C_1^{AZ} &= \frac{c_w \lambda_{HV}}{24 s_w}, \\
C_2^{AZ} &= -\frac{c_w}{8 s_w}\left[\frac{(1+2\lambda_{HV})\,p^2}{6} - m_W^2\right] + \frac{1}{4c_w}\left\{\sum_l \left[\left(\frac{Q_{L_l} I_{3L_l}}{2s_w} - Q_{L_l}^2 s_w\right)\right.\right. \\
&\quad \left.\left.\left(m_{L_l}^2 - \frac{p^2}{6}\right)\right] + N_c \sum_l \left[\left(\frac{Q_{Q_l} I_{3Q_l}}{2s_w} - Q_{Q_l}^2 s_w\right)\left(m_{Q_l}^2 - \frac{p^2}{6}\right)\right]\right\}, \\
C_1^{ZZ} &= -\frac{c_w^2 \lambda_{HV}}{24 s_w^2}, \\
C_2^{ZZ} &= \frac{c_w^2}{8 s_w^2}\left[\frac{(1+2\lambda_{HV})\,p^2}{6} - m_W^2\right] + \frac{1}{4c_w^2}\left\{\sum_l \left[\left(Q_{L_l} I_{3L_l} - \frac{1+g5s}{2}\ \frac{I_{3L_l}^2}{2s_w^2}\right.\right.\right. \\
&\quad \left. - Q_{L_l}^2 s_w^2\right)\left(m_{L_l}^2 - \frac{p^2}{6}\right)\right] + N_c \sum_l \left[\left(Q_{Q_l} I_{Q_l} - \frac{1+g5s}{2}\ \frac{I_{3Q_l}^2}{2s_w^2} - Q_{Q_l}^2 s_w^2\right)\right. \\
&\quad \left.\left.\left(m_{Q_l}^2 - \frac{p^2}{6}\right)\right]\right\}, \\
C_1^{W^-W^+} &= -\frac{\lambda_{HV}}{24 s_w^2},
\end{aligned}
$$

$$
\begin{aligned}
C_2^{W^-W^+} &= \frac{1}{8s_w^2}\left[\frac{(1+2\lambda_{HV})\,p^2}{6} - m_W^2\right] - \frac{1+g5s}{2}\ \frac{1}{32 s_w^2}\left\{\sum_l \left(m_{e_l}^2 - \frac{p^2}{3}\right)\right. \\
&\quad \left. + N_c \sum_{l,m}\left[V_{U_l,D_m} V_{D_m,U_l}^\dagger\left(m_{U_l}^2 + m_{D_m}^2 - \frac{p^2}{3}\right)\right]\right\}. \qquad (4.21)
\end{aligned}
$$

Fermion-Fermion vertices

The generic effective vertex is shown in Fig.4 (c) with the following expression

$$
\mathrm{Vert}\left(F_1, \bar{F}_2\right) = \frac{ie^2}{\pi^2}\ \left[\left(C_-^{F_1\bar{F}_2}\Omega^- + C_+^{F_1\bar{F}_2}\Omega^+\right)\not{p} + C_0^{F_1\bar{F}_2}\right]\ \lambda_{HV}, \qquad (4.22)
$$

with the actual values of F_1,$\bar{F}_2$, $C_-^{F_1\bar{F}_2}$, $C_+^{F_1\bar{F}_2}$ and $C_0^{F_1\bar{F}_2}$

$$
\begin{aligned}
C_-^{U_l\bar{U}_m} &= \delta_{lm}\ \frac{Q_{U_l}^2}{16c_w^2},\\
C_+^{U_l\bar{U}_m} &= \delta_{lm}\ \frac{1}{16}\left[\frac{1+g5s}{2}\ \frac{I_{3U_l}^2}{s_w^2c_w^2} - \frac{1+g5s}{2}\ \frac{2Q_{U_l}I_{3U_l}}{c_w^2} + \frac{Q_{U_l}^2}{c_w^2}\right.\\
&\qquad \left. +\frac{1+g5s}{2}\ \frac{1}{2s_w^2}\sum_g \left(V_{U_l,D_g}V^\dagger_{D_g,U_l}\right)\right],\\
C_0^{U_l\bar{U}_m} &= \delta_{lm}\ \frac{m_{U_l}Q_{U_l}}{8c_w^2}\left(Q_{U_l} - \frac{1+g5s}{2}\ I_{3U_l}\right),\\
C_-^{D_l\bar{D}_m} &= \delta_{lm}\ \frac{Q_{D_l}^2}{16c_w^2},\\
C_+^{D_l\bar{D}_m} &= \delta_{lm}\ \frac{1}{16}\left[\frac{1+g5s}{2}\ \frac{I_{3D_l}^2}{s_w^2c_w^2} - \frac{1+g5s}{2}\ \frac{2Q_{D_l}I_{3D_l}}{c_w^2} + \frac{Q_{D_l}^2}{c_w^2}\right.\\
&\qquad \left. +\frac{1+g5s}{2}\ \frac{1}{2s_w^2}\sum_g \left(V_{U_g,D_l}V^\dagger_{D_l,U_g}\right)\right],\\
C_0^{D_l\bar{D}_m} &= \delta_{lm}\ \frac{m_{D_l}Q_{D_l}}{8c_w^2}\left(Q_{D_l} - \frac{1+g5s}{2}\ I_{3D_l}\right),
\end{aligned}
$$

$$
\begin{aligned}
C_-^{L_l\bar{L}_m} &= \delta_{lm}\ \frac{Q_{L_l}^2}{16c_w^2},\\
C_+^{L_l\bar{L}_m} &= \delta_{lm}\ \frac{1}{16}\left(\frac{1+g5s}{2}\ \frac{I_{3L_l}^2}{s_w^2c_w^2} - \frac{1+g5s}{2}\ \frac{2Q_{L_l}I_{3L_l}}{c_w^2} + \frac{Q_{L_l}^2}{c_w^2}\right.\\
&\qquad \left. +\frac{1+g5s}{2}\ \frac{1}{2s_w^2}\right),\\
C_0^{L_l\bar{L}_m} &= \delta_{lm}\ \frac{m_{L_l}Q_{L_l}}{8c_w^2}\left(Q_{L_l} - \frac{1+g5s}{2}\ I_{3L_l}\right).
\end{aligned}
\tag{4.23}
$$

4.2.2 Electroweak effective vertices with 3 external legs

All possible non-vanishing 3-point vertices in EW are shown in Fig.5.

Scalar-Fermion-Fermion vertices

The generic effective vertex is shown in Fig.5 (a) with the following expression

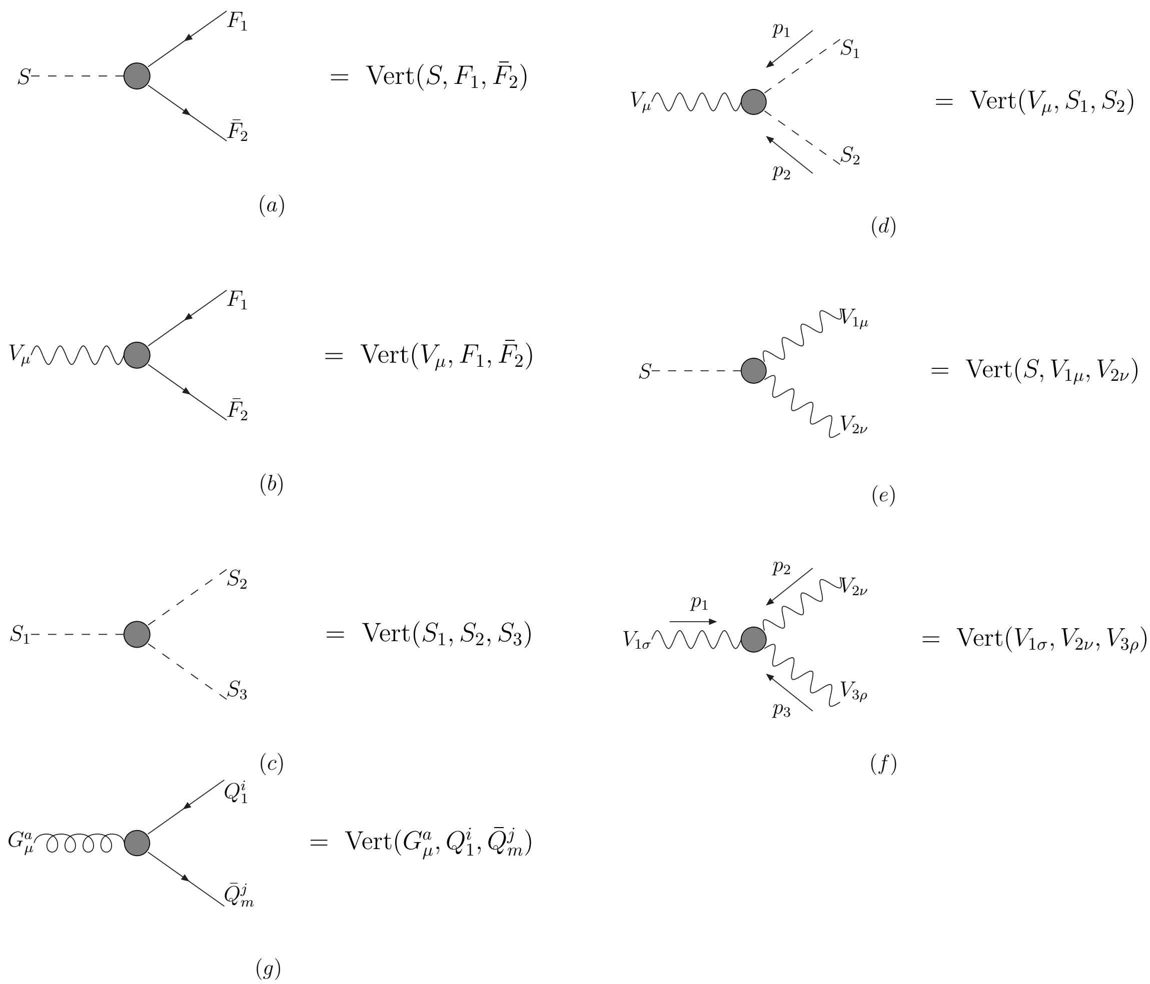


Figure 5: All possible non-vanishing 3-point vertices in EW.

$$\mathrm{Vert}\left(S, F_1, \bar{F}_2\right) = \frac{e^3}{\pi^2}\ \left(C_-^{SF_1\bar{F}_2}\Omega^- + C_+^{SF_1\bar{F}_2}\Omega^+\right), \tag{4.24}$$

With its actual values of S, F_1, $\bar{F}_2$, $C_-^{SF_1\bar{F}_2}$ and $C_+^{SF_1\bar{F}_2}$

$$
\begin{aligned}
C_-^{HU_l\bar{U}_m} &= \delta_{lm}\ \frac{im_{U_l}}{8m_W s_w}\Bigg[\frac{(5+3g5s+8\lambda_{HV})\,Q_{U_l}^2}{16c_w^2}+\frac{1+g5s}{2}\ \frac{1}{16s_w^2} \\
&\qquad \sum_g\Big(V_{U_l,D_g}V_{D_g,U_l}^\dagger\Big)+\frac{1+g5s}{2}\ \frac{I_{3U_l}}{c_w^2}\left(\frac{I_{3U_l}}{8s_w^2}-\frac{(1+\lambda_{HV})\,Q_{U_l}}{2}\right) \\
&\qquad +\frac{1}{16m_W^2 s_w^2}\sum_g\Big(m_{D_g}^2 V_{U_l,D_g}V_{D_g,U_l}^\dagger\Big)\Bigg], \\
C_+^{HU_l\bar{U}_m} &= C_-^{HU_l\bar{U}_m}, \\
C_-^{HD_l\bar{D}_m} &= \delta_{lm}\ \frac{im_{D_l}}{8m_W s_w}\Bigg[\frac{(1+3g5s+4\lambda_{HV})\,Q_{D_l}^2}{8c_w^2}+\frac{1+g5s}{2}\ \frac{1}{16s_w^2} \\
&\qquad \sum_g\Big(V_{U_g,D_l}V_{D_l,U_g}^\dagger\Big)+\frac{1+g5s}{2}\ \frac{I_{3D_l}}{c_w^2}\left(\frac{I_{3D_l}}{8s_w^2}-\frac{(1+\lambda_{HV})\,Q_{D_l}}{2}\right) \\
&\qquad +\frac{1}{16m_W^2 s_w^2}\sum_g\Big(m_{U_g}^2 V_{U_g,D_l}V_{D_l,U_g}^\dagger\Big)\Bigg], \\
C_+^{HD_l\bar{D}_m} &= C_-^{HD_l\bar{D}_m}, \\
C_-^{HL_l\bar{L}_m} &= \delta_{lm}\ \frac{im_{L_l}}{8m_W s_w}\Bigg[\frac{(3+g5s+4\lambda_{HV})\,Q_{L_l}^2}{8c_w^2}+\frac{1+g5s}{2}\ \frac{1}{16s_w^2} \\
&\qquad +\frac{1+g5s}{2}\ \frac{I_{3L_l}}{c_w^2}\left(\frac{I_{3L_l}}{8s_w^2}-\frac{(1+\lambda_{HV})\,Q_{L_l}}{2}\right)\Bigg], \\
C_+^{HL_l\bar{L}_m} &= C_-^{HL_l\bar{L}_m}, \\
C_-^{\phi^0U_l\bar{U}_m} &= -g5s\ \delta_{lm}\ \frac{m_{U_l}}{4m_W s_w}\Bigg[\frac{(5+3g5s+8\lambda_{HV})\,Q_{U_l}^2 I_{3U_l}}{16c_w^2}+\frac{1+g5s}{2}\ \frac{1}{32s_w^2} \\
&\qquad \sum_g\Big(V_{U_l,D_g}V_{D_g,U_l}^\dagger\Big)+\frac{1+g5s}{2}\ \frac{I_{3U_l}}{c_w^2}\left(\frac{1}{32s_w^2}-\frac{(1+\lambda_{HV})\,Q_{U_l}I_{3U_l}}{2}\right) \\
&\qquad -\frac{1}{16m_W^2 s_w^2}\sum_g\Big(m_{D_g}^2 I_{3D_g}V_{U_l,D_g}V_{D_g,U_l}^\dagger\Big)\Bigg], \\
C_+^{\phi^0U_l\bar{U}_m} &= -C_-^{\phi^0U_l\bar{U}_m},
\end{aligned}
$$

$$
\begin{aligned}
C_{-}^{\phi^0 D_l \bar{D}_m} &= -g5s\ \delta_{lm}\ \frac{m_{D_l}}{4 m_W s_w} \left[\frac{(1+3g5s+4\lambda_{HV})\, Q_{D_l}^2 I_{3D_l}}{8c_w^2} - \frac{1+g5s}{2}\ \frac{1}{32 s_w^2} \right. \\
&\quad \sum_g \left(V_{U_g,D_l} V_{D_l,U_g}^{\dagger} \right) + \frac{1+g5s}{2}\ \frac{I_{3D_l}}{c_w^2} \left(\frac{1}{32 s_w^2} - \frac{(1+\lambda_{HV})\, Q_{D_l} I_{3D_l}}{2} \right) \\
&\quad \left. - \frac{1}{16 m_W^2 s_w^2} \sum_g \left(m_{U_g}^2 I_{3U_g} V_{U_g,D_l} V_{D_l,U_g}^{\dagger} \right) \right], \\
C_{+}^{\phi^0 D_l \bar{D}_m} &= -C_{-}^{\phi^0 D_l \bar{D}_m}, \\
C_{-}^{\phi^0 L_l \bar{L}_m} &= -g5s\ \delta_{lm}\ \frac{m_{L_l}}{4 m_W s_w} \left[\frac{(3+g5s+4\lambda_{HV})\, Q_{L_l}^2 I_{3L_1}}{8c_w^2} - \frac{1+g5s}{2}\ \frac{1}{32 s_w^2} \right. \\
&\quad \left. + \frac{1+g5s}{2}\ \frac{I_{3L_l}}{c_w^2} \left(\frac{1}{32 s_w^2} - \frac{(1+\lambda_{HV})\, Q_{L_l} I_{3L_l}}{2} \right) \right], \\
C_{+}^{\phi^0 L_l \bar{L}_m} &= -C_{-}^{\phi^0 L_l \bar{L}_m}, \\
C_{-}^{\phi^- U_l \bar{D}_m} &= -\frac{1+g5s}{2}\ \frac{i m_{D_m} V_{D_m,U_l}^{\dagger}}{4\sqrt{2} m_W s_w} \left[-\frac{1}{16 c_w^2} \left(1 + 8\,(1+\lambda_{HV})\, Q_{U_l} Q_{D_m}\right) - \frac{3}{32 s_w^2} \right. \\
&\quad \left. - \frac{m_{U_l}^2}{16 m_W^2 s_w^2} + \frac{I_{3U_l}}{16 c_w^2} \left(1 + 8\,(1+\lambda_{HV})\, Q_{D_m}\right) \right] + \frac{1-g5s}{2}\ \frac{i m_{U_l} V_{D_m,U_l}^{\dagger}}{4\sqrt{2} m_W s_w} \\
&\quad \left[-\frac{1}{16 c_w^2} \left(1 + 8\,(1+\lambda_{HV})\, Q_{U_l} Q_{D_m}\right) - \frac{m_{D_m}^2}{16 m_W^2 s_w^2} + \frac{3 m_{D_m} - m_{U_l}}{48 m_{U_l} c_w^2} \right], \\
C_{+}^{\phi^- U_l \bar{D}_m} &= \frac{1+g5s}{2}\ \frac{i m_{U_l} V_{D_m,U_l}^{\dagger}}{4\sqrt{2} m_W s_w} \left[-\frac{1}{16 c_w^2} \left(1 + 8\,(1+\lambda_{HV})\, Q_{U_l} Q_{D_m}\right) - \frac{3}{32 s_w^2} \right. \\
&\quad \left. - \frac{m_{D_m}^2}{16 m_W^2 s_w^2} + \frac{I_{3D_m}}{16 c_w^2} \left(1 + 8\,(-1+\lambda_{HV})\, Q_{U_l}\right) \right] - \frac{1-g5s}{2}\ \frac{i m_{D_m} V_{D_m,U_l}^{\dagger}}{4\sqrt{2} m_W s_w} \\
&\quad \left[-\frac{1}{16 c_w^2} \left(1 + 8\,(1+\lambda_{HV})\, Q_{U_l} Q_{D_m}\right) - \frac{m_{U_l}^2}{16 m_W^2 s_w^2} - \frac{5 m_{D_m} - 3 m_{U_l}}{48 m_{D_m} c_w^2} \right], \\
C_{\pm}^{\phi^+ D_l \bar{U}_m} &= -\left(C_{\mp}^{\phi^- U_m \bar{D}_l} \right)^*, \\
C_{-}^{\phi^- \nu_l \bar{e}_m} &= -\delta_{lm}\ \frac{i m_{e_l}}{4\sqrt{2} m_W s_w} \left[\frac{Q_{e_l}}{16 c_w^2} - \frac{1+g5s}{2}\ \frac{3}{32 s_w^2} \right. \\
&\quad \left. + \frac{1+g5s}{2}\ \frac{I_{3\nu_l}}{16 c_w^2} \left(1 + 8\,(1+\lambda_{HV})\, Q_{e_l}\right) \right], \\
C_{+}^{\phi^- \nu_l \bar{e}_m} &= 0, \\
C_{\pm}^{\phi^+ e_l \bar{\nu}_m} &= -\left(C_{\mp}^{\phi^- \nu_m \bar{e}_l} \right)^*.
\end{aligned}
\tag{4.25}
$$

Vector-Fermion-Fermion vertices

The generic effective vertex is shown in Fig.5 (b) with the following expression

$$\mathrm{Vert}\left(V_\mu, F_1, \bar{F}_2\right) = \frac{ie^3}{\pi^2} \left(C_-^{VF_1\bar{F}_2}\Omega^- + C_+^{VF_1\bar{F}_2}\Omega^+\right) \gamma_\mu, \tag{4.26}$$

with the actual values of V, F_1, $\bar{F}_2$, $C_-^{VF_1\bar{F}_2}$ and $C_+^{VF_1\bar{F}_2}$

$$\begin{aligned}
C_-^{AU_l\bar{U}_m} &= \delta_{lm} \frac{1}{4} \left[\frac{1+\lambda_{HV}}{4c_w^2} Q_{U_l}^3 + \frac{m_{U_l}^2}{8s_w^2 m_W^2} \left(\frac{1}{2} \sum_g \left(V_{U_l,D_g} V_{D_g,U_l}^\dagger Q_{D_g} \right) \right.\right. \\
&\quad \left.\left. + \frac{Q_{U_l}}{4} + Q_{U_l} I_{3U_l}^2 \right) \right], \\
C_+^{AU_l\bar{U}_m} &= \delta_{lm} \frac{1}{4} \left\{ \frac{1+\lambda_{HV}}{4c_w^2} Q_{U_l}^3 - \frac{2+(1+g5s)\ \lambda_{HV}}{4c_w^2} Q_{U_l}^2 I_{3U_l} \right. \\
&\quad + \frac{2+(1+g5s)\,\lambda_{HV}}{8s_w^2 c_w^2} Q_{U_l} I_{3U_l}^2 + \frac{1}{4s_w^2} \left[\frac{1}{4m_W^2} \sum_g \left(V_{U_l,D_g} V_{D_g,U_l}^\dagger m_{D_g}^2 Q_{D_g} \right) \right. \\
&\quad + \frac{1+4I_{3U_l}^2}{8m_W^2} m_{U_l}^2 Q_{U_l} + \frac{2+(1+g5s)\,\lambda_{HV}}{4} \\
&\quad \left.\left. \sum_g \left(V_{U_l,D_g} V_{D_g,U_l}^\dagger \left(\frac{1+g5s}{2} + Q_{D_g} \right) \right) \right] \right\}, \\
C_-^{AD_l\bar{D}_m} &= \delta_{lm} \frac{1}{4} \left[\frac{1+\lambda_{HV}}{4c_w^2} Q_{D_l}^3 + \frac{m_{D_l}^2}{8s_w^2 m_W^2} \left(\frac{1}{2} \sum_g \left(V_{U_g,D_l} V_{D_l,U_g}^\dagger Q_{U_g} \right) \right.\right. \\
&\quad \left.\left. + \frac{Q_{D_l}}{4} + Q_{D_l} I_{3D_l}^2 \right) \right], \\
C_+^{AD_l\bar{D}_m} &= \delta_{lm} \frac{1}{4} \left\{ \frac{1+\lambda_{HV}}{4c_w^2} Q_{D_l}^3 - \frac{2+(1+g5s)\ \lambda_{HV}}{4c_w^2} Q_{D_l}^2 I_{3D_l} \right. \\
&\quad + \frac{2+(1+g5s)\,\lambda_{HV}}{8s_w^2 c_w^2} Q_{D_l} I_{3D_l}^2 + \frac{1}{4s_w^2} \left[\frac{1}{4m_W^2} \sum_g \left(V_{U_g,D_l} V_{D_l,U_g}^\dagger m_{U_g}^2 Q_{U_g} \right) \right. \\
&\quad + \frac{1+4I_{3D_l}^2}{8m_W^2} m_{D_l}^2 Q_{D_l} + \frac{2+(1+g5s)\,\lambda_{HV}}{4} \\
&\quad \left.\left. \sum_g \left(V_{U_l,D_g} V_{D_g,U_l}^\dagger \left(Q_{U_g} - \frac{1+g5s}{2} \right) \right) \right] \right\}, \\
C_-^{Ae_l\bar{e}_m} &= \delta_{lm} \frac{1}{4} \left[\frac{1+\lambda_{HV}}{4c_w^2} Q_{e_l}^3 + \frac{m_{e_l}^2}{8m_W^2 s_w^2} \left(\frac{Q_{e_l}}{4} + Q_{e_l} I_{3e_l}^2 \right) \right],
\end{aligned}$$

$$
\begin{aligned}
C_+^{Ae_l\bar{e}_m} &= \delta_{lm}\ \frac{1}{4}\left[\frac{1+\lambda_{HV}}{4c_w^2}Q_{e_l}^3 - \frac{2+(1+g5s)\,\lambda_{HV}}{4c_w^2}Q_{e_l}^2 I_{3e_l}\right.\\
&\quad +\frac{2+(1+g5s)\,\lambda_{HV}}{8s_w^2c_w^2}Q_{e_l}I_{3e_l}^2 + \frac{1}{4s_w^2}\left(\frac{1+4I_{3e_l}^2}{8m_W^2}m_{e_l}^2Q_{e_l}\right.\\
&\quad \left.\left. -\frac{1+g5s}{2}\ \frac{1+\lambda_{HV}}{2}\right)\right],\\
C_-^{A\nu_l\bar{\nu}_m} &= 0,\\
C_+^{A\nu_l\bar{\nu}_m} &= \delta_{lm}\ \frac{1}{32s_w^2}\left[\frac{m_{e_l}^2Q_{e_l}}{2m_W^2} + \left(Q_{e_l}+\frac{1+g5s}{2}\right)\left(\frac{1+g5s}{2}\lambda_{HV}+1\right)\right],\\
C_-^{ZU_l\bar{U}_m} &= \delta_{lm}\ \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,s_w}{2c_w^2}Q_{U_l}^3 + \frac{m_{U_l}^2}{8s_wm_W^2}\left[\sum_g\left(V_{U_l,D_g}V_{D_g,U_l}^\dagger\right.\right.\right.\\
&\quad \left.\left.\left(Q_{D_g}-\frac{1+g5s}{2}\ \frac{I_{3D_g}}{s_w^2}\right)\right)+Q_{U_l}-\frac{1+g5s}{2}\ \frac{I_{3U_l}}{s_w^2}\right]\\
&\quad \left. -\frac{1-g5s}{2}\frac{1+\lambda_{HV}}{2s_wc_w^2}Q_{U_l}^2I_{3U_l}\right\},\\
C_+^{ZU_l\bar{U}_m} &= \delta_{lm}\ \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,s_w}{2c_w^2}Q_{U_l}^3 - \frac{(2+(1+g5s)\ \lambda_{HV})\,(4s_w^2+1+g5s)}{8c_w^2s_w}\right.\\
&\quad Q_{U_l}^2I_{3U_l} + (2+g5s)\,\frac{2+(1+g5s)\,\lambda_{HV}}{4s_wc_w^2}Q_{U_l}I_{3U_l}^2\\
&\quad -\frac{1+g5s}{2}\ \frac{(1+\lambda_{HV})\,I_{3U_l}^3}{2s_w^3c_w^2} - \frac{1-g5s}{2}\ \frac{m_{U_l}^2I_{3U_l}}{8m_W^2s_w^3} + \frac{1}{2s_w}\left[\frac{1}{4m_W^2}\right.\\
&\quad \sum_g\left(V_{U_l,D_g}V_{D_g,U_l}^\dagger m_{D_g}^2\left(Q_{D_g}-\frac{1-g5s}{2}\ \frac{I_{3D_g}}{s_w^2}\right)\right)\\
&\quad +\frac{1+4I_{3U_l}^2}{8m_W^2}m_{U_l}^2Q_{U_l} + \frac{2+(1+g5s)\,\lambda_{HV}}{4}\\
&\quad \left.\left.\sum_g\left(V_{U_l,D_g}V_{D_g,U_l}^\dagger\left(Q_{D_g}-\frac{1+g5s}{2}\ \frac{c_w^2+I_{3D_g}}{s_w^2}\right)\right)\right]\right\},\\
C_-^{ZD_l\bar{D}_m} &= \delta_{lm}\ \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,s_w}{2c_w^2}Q_{D_l}^3 + \frac{m_{D_l}^2}{8s_wm_W^2}\left[\sum_g\left(V_{U_g,D_l}V_{D_l,U_g}^\dagger\right.\right.\right.\\
&\quad \left.\left.\left(Q_{U_g}-\frac{1+g5s}{2}\ \frac{I_{3U_g}}{s_w^2}\right)\right)+Q_{D_l}-\frac{1+g5s}{2}\ \frac{I_{3D_l}}{s_w^2}\right]\\
&\quad \left. -\frac{1-g5s}{2}\frac{1+\lambda_{HV}}{2s_wc_w^2}Q_{D_l}^2I_{3D_l}\right\},
\end{aligned}
$$

$$
\begin{aligned}
C_{+}^{ZD_l\bar{D}_m} &= \delta_{lm}\ \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,s_w}{2c_w^2}Q_{D_l}^3 - \frac{(2+(1+g5s)\ \ \lambda_{HV})\,(4s_w^2+1+g5s)}{8c_w^2 s_w}\right.\\
&\quad Q_{D_l}^2 I_{3D_l} + (2+g5s)\,\frac{2+(1+g5s)\,\lambda_{HV}}{4s_w c_w^2}Q_{D_l}I_{3D_l}^2\\
&\quad -\frac{1+g5s}{2}\ \frac{(1+\lambda_{HV})\,I_{3D_l}^3}{2s_w^3 c_w^2} - \frac{1-g5s}{2}\ \frac{m_{D_l}^2 I_{3D_l}}{8m_W^2 s_w^3} + \frac{1}{2s_w}\left[\frac{1}{4m_W^2}\right.\\
&\quad \sum_g\left(V_{U_g,D_l}V_{D_l,U_g}^{\dagger}m_{U_g}^2\left(Q_{U_g} - \frac{1-g5s}{2}\ \frac{I_{3U_g}}{s_w^2}\right)\right)\\
&\quad +\frac{1+4I_{3D_l}^2}{8m_W^2}m_{D_l}^2 Q_{D_l} + \frac{2+(1+g5s)\,\lambda_{HV}}{4}\\
&\quad \left.\left.\sum_g\left(V_{U_g,D_l}V_{D_l,U_g}^{\dagger}\left(Q_{U_g} + \frac{1+g5s}{2}\ \frac{c_w^2 - I_{3U_g}}{s_w^2}\right)\right)\right]\right\},\\
C_{-}^{Ze_l\bar{e}_m} &= \delta_{lm}\ \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,s_w}{2c_w^2}Q_{e_l}^3 + \frac{m_{e_l}^2}{4s_w m_W^2}\left[-\frac{1+g5s}{2}\ \frac{1}{4s_w^2} + \frac{1}{2}\right.\right.\\
&\quad \left.\left.\left(Q_{e_l} - \frac{1+g5s}{2}\ \frac{I_{3e_l}}{s_w^2}\right)\right] - \frac{1-g5s}{2}\frac{1+\lambda_{HV}}{2s_w c_w^2}Q_{e_l}^2 I_{3e_l}\right\},\\
C_{+}^{Ze_l\bar{e}_m} &= \delta_{lm}\ \frac{1}{16c_w}\left\{\frac{(1+\lambda_{HV})\,s_w}{c_w^2}Q_{e_l}^3 - \frac{(2+(1+g5s)\ \ \lambda_{HV})\,(4s_w^2+1+g5s)}{4c_w^2 s_w}\right.\\
&\quad Q_{e_l}^2 I_{3e_l} + (2+g5s)\,\frac{2+(1+g5s)\,\lambda_{HV}}{2s_w c_w^2}Q_{e_l}I_{3e_l}^2\\
&\quad -\frac{1+g5s}{2}\ \frac{(1+\lambda_{HV})\,I_{3e_l}^3}{s_w^3 c_w^2} - \frac{1-g5s}{2}\ \frac{m_{e_l}^2 I_{3e_l}}{4m_W^2 s_w^3}\\
&\quad \left.+\frac{1}{2s_w}\left[\frac{m_{e_l}^2 Q_{e_l}\left(1+4I_{3e_l}^2\right)}{4m_W^2} + \frac{1+g5s}{2}\ \frac{(1+\lambda_{HV})\,(c_w^2 - I_{3\nu_l})}{s_w^2}\right]\right\},\\
C_{-}^{Z\nu_l\bar{\nu}_m} &= 0,\\
C_{+}^{Z\nu_l\bar{\nu}_m} &= \delta_{lm}\ \frac{1}{16c_w}\left\{-\frac{1+g5s}{2}\ \frac{(1+\lambda_{HV})\,I_{3\nu_l}^3}{s_w^3 c_w^2} + \frac{1}{2s_w}\left[\frac{m_{e_l}^2 Q_{e_l}}{2m_W^2}\right.\right.\\
&\quad \left.+\left(1+\frac{1+g5s}{2}\lambda_{HV}\right)\left(Q_{e_l} - \frac{1+g5s}{2}\ \frac{c_w^2 + I_{3e_l}}{s_w^2}\right)\right]\\
&\quad \left.-\frac{1-g5s}{2}\ \frac{I_{3e_l}m_{e_l}^2}{4m_W^2 s_w^3}\right\},
\end{aligned}
$$

$$
\begin{aligned}
C_{-}^{W^{-}U_l\bar{D}_m} &= -\frac{1-g5s}{2}\ \frac{V^{\dagger}_{Dm,U_l}}{16\sqrt{2}s_w}\left(1+\lambda_{HV}\right)\frac{Q_{U_l}Q_{D_m}}{c_w^2},\\
C_{+}^{W^{-}U_l\bar{D}_m} &= \frac{1+g5s}{2}\ \frac{V^{\dagger}_{Dm,U_l}}{16\sqrt{2}s_w}\left(1+\lambda_{HV}\right)\left[\frac{Q_{D_m}I_{3U_l}+Q_{U_l}I_{3D_m}-Q_{U_l}Q_{D_m}}{c_w^2}\right.\\
&\left.-\frac{1}{s_w^2}+\frac{1}{4s_w^2c_w^2}\right],\\
C_{\pm}^{W^{+}D_l\bar{U}_m} &= \left(C_{\pm}^{W^{-}U_m\bar{D}_l}\right)^{*},\\
C_{-}^{W^{-}\nu_l\bar{e}_m} &= 0,\\
C_{+}^{W^{-}\nu_l\bar{e}_m} &= \frac{1+g5s}{2}\ \delta_{lm}\ \frac{1}{16\sqrt{2}s_w}\left[\frac{Q_{e_l}I_{3\nu_l}}{c_w^2}-\frac{1}{s_w^2}+\frac{1}{4s_w^2c_w^2}\right]\left(1+\lambda_{HV}\right),\\
C_{\pm}^{W^{+}e_l\bar{\nu}_m} &= \left(C_{\pm}^{W^{-}\nu_m\bar{e}_l}\right)^{*}.
\end{aligned}
\tag{4.27}
$$

Scalar-Scalar-Scalar vertices

The generic effective vertex is shown in Fig.5 (c) with the following expression

$$
\mathrm{Vert}\left(S_1,S_2,S_3\right)=\frac{ie^3}{\pi^2}\ C^{S_1S_2S_3},
\tag{4.28}
$$

with its actual values of S_1, S_2, S_3 and $C^{S_1S_2S_3}$

$$
\begin{aligned}
C^{HH\phi^0} &= 0,\qquad C^{\phi^0\phi^0\phi^0}=0,\qquad C^{\phi^0\phi^-\phi^+}=0,\\
C^{HHH} &= \frac{3}{32s_w^2}\left[\frac{1-4\lambda_{HV}}{2}m_W+\frac{1}{m_W^3}\left(\sum_l m_{e_l}^4+N_c\sum_l m_{Q_l}^4\right)\right.\\
&\left.+\left(1+\frac{1}{2c_w^2}\right)\frac{m_H^2}{4m_W}+\frac{\left(1-4\lambda_{HV}\right)m_W}{4c_w^4}\right],\\
C^{H\phi^0\phi^0} &= \frac{1}{8s_w^3}\left[\frac{1-4\lambda_{HV}}{8}m_W+\frac{2g5s-1}{4m_W^3}\left(\sum_l m_{e_l}^4+N_c\sum_l m_{Q_l}^4\right)\right.\\
&\left.+\left(1+\frac{1}{2c_w^2}\right)\frac{m_H^2}{4m_W}+\frac{\left(1-4\lambda_{HV}\right)m_W}{16c_w^4}\right],
\end{aligned}
$$

$$
\begin{aligned}
C^{H\phi^-\phi^+} &= \frac{1}{32s_w^3}\left\{\frac{1-4\lambda_{HV}}{4}m_W\left(3+\frac{s_w^2\left(1+c_w^2\right)}{c_w^4}\right)+\frac{\left(1+2c_w^2\right)m_H^2}{8c_w^2 m_W}\right. \\
&\quad \frac{1}{m_W^3}\left[\frac{1+g5s}{2}\sum_l m_{e_l}^4+N_c\sum_{l,m}\left[V_{U_l,D_m}V^\dagger_{D_m,U_l}\right.\right. \\
&\quad \left(\frac{1+g5s}{2}\left(m_{U_l}^4+m_{D_m}^4\right)-\frac{1-g5s}{2}\,2m_{U_l}m_{D_m}\right. \\
&\quad \left.\left.\left.\left.\left(m_{U_l}^2+m_{U_l}m_{D_m}+m_{D_m}^2\right)\right)\right]\right]\right\}. \qquad (4.29)
\end{aligned}
$$

Vector-Scalar-Scalar vertices

The generic effective vertex is shown in Fig.5 (d) with the following expression

$$
\mathrm{Vert}\left(V_\mu,S_1,S_2\right)=\frac{e^3}{\pi^2}\,C_\mu^{VS_1S_2}, \qquad (4.30)
$$

with the actual values of V, S_1, S_2 and $C_\mu^{VS_1S_2}$

$$
\begin{aligned}
C_\mu^{AHH} &= 0,\quad C_\mu^{ZHH}=0,\quad C_\mu^{A\phi^0\phi^0}=0,\quad C_\mu^{Z\phi^0\phi^0}=0, \\
C_\mu^{A\phi^0 H} &= \frac{5}{192s_w^2}\left(p_1-p_2\right)_\mu, \\
C_\mu^{Z\phi^0 H} &= -\frac{1}{96s_wc_w}\left[\frac{1+2c_w^2+20c_w^4}{8s_w^2c_w^2}+\frac{1}{s_w^2m_W^2}\left(\sum_l m_{e_l}^2+N_c\sum_l m_{Q_l}^2\right)\right] \\
&\quad \left(p_1-p_2\right)_\mu-\frac{1-g5s}{2}\,\frac{3}{96s_wc_w}\left[\frac{1}{s_w^2m_W^2}\left(\sum_l m_{e_l}^2+N_c\sum_l m_{Q_l}^2\right)\right]\left(p_2\right)_\mu, \\
C_\mu^{A\phi^+\phi^-} &= g5s\,\frac{i}{48s_w^2}\left[\frac{1+12c_w^2}{8c_w^2}+\frac{3+g5s}{4m_W^2}\left(\sum_l\left(m_{e_l}^2\right)+N_c\sum_{l,m}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right.\right.\right. \\
&\quad \left.\left.\left.\left.\left(m_{U_l}^2+m_{D_m}^2-\frac{1-g5s}{2}\,2m_{U_l}m_{D_m}\right)\right)\right)\right]\left(p_1-p_2\right)_\mu, \\
C_\mu^{Z\phi^+\phi^-} &= g5s\,\frac{i}{48s_wc_w}\left\{\frac{1-24c_w^4}{16c_w^2s_w^2}+\frac{3+g5s}{4m_W^2}\left[\sum_l\left(m_{e_l}^2\left(1-\frac{1}{2s_w^2}\right)\right)\right.\right. \\
&\quad +N_c\sum_{l,m}\left[V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}^2+m_{D_m}^2-\frac{1-g5s}{2}\,2m_{U_l}m_{D_m}\right)\right. \\
&\quad \left.\left.\left.\left(1-\frac{1}{2s_w^2}\right)\right]\right]\right\}\left(p_1-p_2\right)_\mu,
\end{aligned}
$$

$$
\begin{aligned}
C_\mu^{W^- H\phi^+} &= \frac{i}{96 s_w^3}\left\{\frac{1+22c_w^2}{8c_w^2}+\frac{1}{m_W^2}\left[\sum_l m_{e_l}^2 + N_c\sum_{l,m}\Big(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right.\right.\\
&\quad \left.\left.\left(m_{U_l}^2+m_{D_m}^2\right)\Big)\right]\right\}\ \left(p_1-p_2\right)_\mu - \frac{1-g5s}{2}\ \frac{i}{96s_w^3}\frac{3}{2m_W^2}\\
&\quad \left[\sum_l m_{e_l}^2 + N_c\sum_{l,m}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}+m_{D_m}\right)^2\right)\right]\ \left(p_1\right)_\mu,\\
C_\mu^{W^+\phi^- H} &= \frac{i}{96 s_w^3}\left\{\frac{1+22c_w^2}{8c_w^2}+\frac{1}{m_W^2}\left[\sum_l m_{e_l}^2 + N_c\sum_{l,m}\Big(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right.\right.\\
&\quad \left.\left.\left(m_{U_l}^2+m_{D_m}^2\right)\Big)\right]\right\}\ \left(p_1-p_2\right)_\mu + \frac{1-g5s}{2}\ \frac{i}{96s_w^3}\frac{3}{2m_W^2}\\
&\quad \left[\sum_l m_{e_l}^2 + N_c\sum_{l,m}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}+m_{D_m}\right)^2\right)\right]\ \left(p_2\right)_\mu,\\
C_\mu^{W^-\phi^+\phi^0} &= \frac{1}{48 s_w^3}\left\{-\frac{1+22c_w^2}{16c_w^2}+\frac{3+g5s}{4m_W^2}\left[\sum_l\left(m_{e_l}^2 I_{3e_l}\right)-\frac{N_c}{2}\sum_{l,m}\Big(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right.\right.\\
&\quad \left.\left.\left(m_{U_l}^2+m_{D_m}^2-\left(1-g5s\right)m_{U_l}m_{D_m}\right)\Big)\right]\right\}\ \left(p_1-p_2\right)_\mu\\
&\quad -\frac{1-g5s}{2}\ \frac{3}{96m_W^2 s_w^3}\left[\sum_l\left(m_{e_l}^2 I_{3e_l}\right)\right.\\
&\quad \left.-\frac{N_c}{2}\sum_{l,m}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}-m_{D_m}\right)^2\right)\right]\left(p_1\right)_\mu,\\
C_\mu^{W^+\phi^-\phi^0} &= C_\mu^{W^-\phi^+\phi^0}.
\end{aligned}
\tag{4.31}
$$

Scalar-Vector-Vector vertices

The generic effective vertex is shown in Fig.5 (e) with the following expression

$$
\mathrm{Vert}\left(S, V_{1\mu}, V_{2\nu}\right) = \frac{ie^3}{\pi^2}\ C^{SV_1V_2}\ g_{\mu\nu}, \tag{4.32}
$$

with the actual values of S, V_1, V_2 and $C^{SV_1V_2}$

$$
\begin{aligned}
C^{\phi^0 AA} &= 0, \quad C^{\phi^0 AZ} = 0, \quad C^{\phi^0 ZZ} = 0, \quad C^{\phi^0 W^- W^+} = 0, \\
C^{HAA} &= -\frac{1}{8s_w}\left[\frac{1}{m_W}\left(\sum_l\left(m_{L_l}^2 Q_{L_l}^2\right) + N_c\sum_l\left(m_{Q_l}^2 Q_{Q_l}^2\right)\right) + \frac{m_W}{2}\right], \\
C^{HAZ} &= \frac{1}{8c_w}\left\{\frac{1}{m_W}\left[\sum_l\left(m_{L_l}^2 Q_{L_l}\left(\frac{I_{3L_l}}{2s_w^2} - Q_{L_l}\right)\right)\right.\right. \\
&\quad \left.\left. + N_c\sum_l\left(m_{Q_l}^2 Q_{Q_l}\left(\frac{I_{3Q_l}}{2s_w^2} - Q_{Q_l}\right)\right)\right] + \frac{m_W^2\left(1+2c_w^2\right)}{4s_w^2}\right\}, \\
C^{HZZ} &= \frac{1}{8}\left\{\frac{1}{m_W c_w^2}\left[\sum_l\left(m_{L_l}^2\left(\frac{Q_{L_l} I_{3L_l}}{s_w} - Q_{L_l}^2 s_w - \frac{1+g5s}{2}\ \frac{I_{3L_l}^2}{s_w^3}\right)\right)\right.\right. \\
&\quad \left. + N_c\sum_l\left(m_{Q_l}^2\left(\frac{Q_{Q_l} I_{3L_l}}{s_w} - Q_{Q_l}^2 s_w - \frac{1+g5s}{2}\ \frac{I_{3Q_l}^2}{s_w^3}\right)\right)\right] \\
&\quad \left. + \frac{m_W\left(s_w^2 - 2\right)}{2s_w^3}\right\}, \\
C^{HW^-W^+} &= -\frac{1}{8s_w^3}\left\{\frac{1+g5s}{2}\ \frac{1}{4m_W}\left[\sum_l m_{L_l}^2\right.\right. \\
&\quad \left.\left. + N_c\sum_{l,m}\left(V_{U_l,D_m} V_{D_m,U_l}^\dagger\left(m_{U_l}^2 + m_{D_m}^2\right)\right)\right] + m_W\right\}, \\
C^{\phi^- AW^+} &= C^{\phi^+ W^- A} = \frac{K}{32s_w^2}, \\
C^{\phi^- ZW^+} &= C^{\phi^+ W^- Z} = \frac{K}{32s_w c_w},
\end{aligned}
\tag{4.33}
$$

where

$$
K = m_W + \frac{N_c}{m_W}\sum_{l,m}\left(V_{U_l,D_m} V_{D_m,U_l}^\dagger\left(Q_{U_l} m_{D_m}^2 - Q_{D_m} m_{U_l}^2\right)\right). \tag{4.34}
$$

Vector-Vector-Vector vertices

The generic effective vertex is shown in Fig.5 (f) with the following expression

$$
\mathrm{Vert}\left(V_{1\mu}, V_{2\nu}, V_{3\rho}\right) = \frac{ie^3}{\pi^2}\ C_{\mu\nu\rho}^{V_1 V_2 V_3}, \tag{4.35}
$$

with the actual values of V_1, V_2, V_3, and $C^{V_1V_2V_3}_{\mu\nu\rho}$

$$
\begin{aligned}
C^{AAA}_{\mu\nu\rho} &= C^{AAZ}_{\mu\nu\rho} = C^{AZZ}_{\mu\nu\rho} = C^{ZZZ}_{\mu\nu\rho} = 0, \\
C^{AW^+W^-}_{\mu\nu\rho} &= K\ V_{1\mu\nu\rho} + \frac{1+3g5s}{2}\ \frac{i}{288 s_w^2}\left[\left(N_c \sum_{l,m=1}^{3}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right) - 9\right)\right. \\
&\quad \left.\left(\varepsilon_{(p_2-p_3)\mu\nu\rho} + \frac{1-g5s}{2}\ 3iV_{2\mu\nu\rho}\right)\right] + \frac{1-g5s}{2}\ \frac{1}{8s_w^2}V_{2\mu\nu\rho}, \\
C^{ZW^+W^-} &= -\frac{c_w}{s_w}\left(K - \frac{1-g5s}{2}\ \frac{1}{4s_w^2}\right)V_{1\mu\nu\rho} + \frac{1-g5s}{2}\ \frac{1}{8c_w s_w}V_{3\mu\nu\rho} \\
&\quad + \frac{1+3g5s}{2}\ \frac{1}{288 c_w s_w^3}\left(N_c \sum_{l,m=1}^{3}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right) - 9\right) \\
&\quad \left(i s_w^2 \varepsilon_{(p_2-p_3)\mu\nu\rho} - \frac{1-g5s}{2}\ 3\left(-2V_{1\mu\nu\rho} + s_w^2 V_{2\mu\nu\rho}\right)\right),
\end{aligned}
\tag{4.36}
$$

where

$$
\begin{aligned}
K &= \frac{7+4\lambda_{HV}}{96 s_w^2} + \frac{1}{48 s_w^2}\left(3 + N_c \sum_{l,m=1}^{3}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\right)\right), \\
V_{1\mu\nu\rho} &= g_{\mu\nu}\left(p_2-p_1\right)_\rho + g_{\nu\rho}\left(p_3-p_2\right)_\mu + g_{\rho\nu}\left(p_1-p_3\right)_\nu, \\
V_{2\mu\nu\rho} &= 3g_{\mu\nu}\left(p_1\right)_\rho + g_{\nu\rho}\left(p_2-p_3\right)_\mu - 3g_{\rho\nu}\left(p_1\right)_\nu, \\
V_{3\mu\nu\rho} &= g_{\mu\nu}\left(p_2-p_3\right)_\rho + g_{\mu\rho}\left(p_2-p_3\right)_\nu - g_{\nu\rho}\left(p_2-p_3\right)_\mu.
\end{aligned}
\tag{4.37}
$$

Gluon-Quark-Quark vertices

In previous subsection, we have listed all mixed R QCD/EW vertices with internal QCD particles. The remaining non-vanishing mixed R QCD/EW vertices with EW internal particles are those Gluon-Quark-Quark vertices. Its generic diagram is shown in Fig.5 (g) with the following expression

$$
\mathrm{Vert}\left(G^a_\mu, Q^i_l, \bar{Q}^j_m\right) = \frac{i g_s e^2}{\pi^2}\ T^a_{ji}\ \left(C^{Q_l\bar{Q}_m}_-\Omega^- + C^{Q_l\bar{Q}_m}_+\Omega^+\right)\gamma_\mu. \tag{4.38}
$$

The actual values of Q_l, $\bar{Q}_m$, $C^{Q_l\bar{Q}_m}_-$ and $C^{Q_l\bar{Q}_m}_+$ are

$$
\begin{aligned}
C_-^{U_l\bar{U}_m} &= \delta_{lm}\ \frac{1}{16}\left[(1+\lambda_{HV})\,\frac{Q_{U_l}^2}{c_w^2}+\frac{m_{U_l}^2}{2s_w^2 m_W^2}\left(\frac{1}{2}\sum_g\left(V_{U_l,D_g}V^\dagger_{D_g,U_l}\right)\right.\right.\\
&\qquad\left.\left.+\frac{1}{4}+I_{3U_l}^2\right)\right],\\
C_+^{U_l\bar{U}_m} &= \delta_{lm}\ \frac{1}{16}\left[(1+\lambda_{HV})\,\frac{Q_{U_l}^2}{c_w^2}+\frac{2+(1+g5s)\,\lambda_{HV}}{2c_w^2}\left(\frac{I_{3U_l}^2}{s_w^2}-2Q_{U_l}I_{3U_l}\right)\right.\\
&\qquad+\frac{2+(1+g5s)\,\lambda_{HV}}{4s_w^2}\sum_g\left(V_{U_l,D_g}V^\dagger_{D_g,U_l}\right)+\frac{1}{2m_W^2 s_w^2}\\
&\qquad\left.\left(\frac{1}{2}\sum_g\left(V_{U_l,D_g}V^\dagger_{D_g,U_l}m_{D_g}^2\right)+m_{U_l}^2\left(\frac{1}{4}+I_{3U_l}^2\right)\right)\right],\\
C_-^{D_l\bar{D}_m} &= \delta_{lm}\ \frac{1}{16}\left[(1+\lambda_{HV})\,\frac{Q_{D_l}^2}{c_w^2}+\frac{m_{D_l}^2}{2s_w^2 m_W^2}\left(\frac{1}{2}\sum_g\left(V_{U_g,D_l}V^\dagger_{D_l,U_g}\right)\right.\right.\\
&\qquad\left.\left.+\frac{1}{4}+I_{3D_l}^2\right)\right],\\
C_+^{D_l\bar{D}_m} &= \delta_{lm}\ \frac{1}{16}\left[(1+\lambda_{HV})\,\frac{Q_{D_l}^2}{c_w^2}+\frac{2+(1+g5s)\,\lambda_{HV}}{2c_w^2}\left(\frac{I_{3D_l}^2}{s_w^2}-2Q_{D_l}I_{3D_l}\right)\right.\\
&\qquad+\frac{2+(1+g5s)\,\lambda_{HV}}{4s_w^2}\sum_g\left(V_{U_g,D_l}V^\dagger_{D_l,U_g}\right)+\frac{1}{2m_W^2 s_w^2}\\
&\qquad\left.\left(\frac{1}{2}\sum_g\left(V_{U_g,D_l}V^\dagger_{D_l,U_g}m_{U_g}^2\right)+m_{D_l}^2\left(\frac{1}{4}+I_{3D_l}^2\right)\right)\right].
\end{aligned}
\tag{4.39}
$$

4.2.3 Electroweak effective vertices with 4 external legs

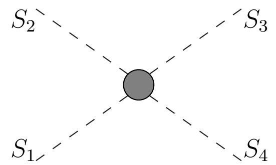
In this part, we list all possible non-vanishing 4-point vertices in EW, which are shown in Fig.6.

Scalar-Scalar-Scalar-Scalar vertices

The generic effective vertex is shown in Fig.6 (a) with the following expression

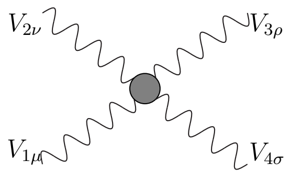
$$
\mathrm{Vert}\,(S_1,S_2,S_3,S_4)=\frac{ie^4}{\pi^2}\,C^{S_1S_2S_3S_4},
\tag{4.40}
$$

with the actual values of S_1, S_2, S_3, S_4 and $C^{S_1S_2S_3S_4}$



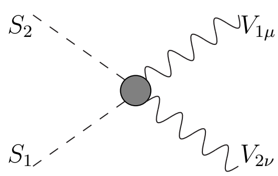
$$= \mathrm{Vert}\left(S_1, S_2, S_3, S_4\right)$$

(a)



$$= \mathrm{Vert}\left(V_{1\mu}, V_{2\nu}, V_{3\rho}, V_{4\sigma}\right)$$

(b)



$$= \mathrm{Vert}\left(S_1, S_2, V_{1\mu}, V_{2\nu}\right)$$

(c)

Figure 6: All possible non-vanishing 4-point vertices in EW.

$$\begin{aligned}
C^{HHH\phi^0} &= C^{H\phi^0\phi^0\phi^0} = C^{H\phi^0\phi^-\phi^+} = 0, \\
C^{HHHH} &= C^{\phi^0\phi^0\phi^0\phi^0} = \frac{1}{64 s_w^4} K_1, \\
C^{HH\phi^0\phi^0} &= \frac{1}{192 s_w^4} \left(K_1 - \frac{1-g5s}{2}\ \frac{4}{m_W^4} \left(\sum_l m_{e_l}^4 + N_c \sum_l m_{Q_l}^4 \right) \right), \\
C^{HH\phi^-\phi^+} &= \frac{1}{64 s_w^4} \left\{ K_2 - \frac{1-g5s}{2}\ \frac{2N_c}{3m_W^4} \right. \\
&\quad \left. \sum_{l,m} \left[V_{U_l,D_m} V^\dagger_{D_m,U_l} m_{U_l} m_{D_m} \left(m_{U_l}^2 + m_{U_l} m_{D_m} + m_{D_m}^2 \right) \right] \right\},
\end{aligned}$$

$$
\begin{aligned}
C^{\phi^0\phi^0\phi^-\phi^+} &= \frac{1}{64s_w^4}\left\{K_2 + \frac{1-g5s}{2}\;\frac{2N_c}{3m_W^4}\right.\\
&\left.\sum_{l,m}\left[V_{U_l,D_m}V^\dagger_{D_m,U_l}m_{U_l}m_{D_m}\left(m^2_{U_l}-m_{U_l}m_{D_m}+m^2_{D_m}\right)\right]\right\},\\
C^{\phi^-\phi^+\phi^-\phi^+} &= \frac{1}{32s_w^4}K_3,
\end{aligned}
\tag{4.41}
$$

where

$$
\begin{aligned}
K_1 &= \frac{1}{m_W^2}\left[\frac{5}{m_W^2}\left(\sum_g m^4_{e_l}+N_c\sum_g m^4_{Q_l}\right)+\frac{3m_H^2}{2}\left(1+\frac{1}{2c_w^2}\right)\right]\\
&\quad+\frac{1-12\lambda_{HV}}{2}\left(1+\frac{1}{2c_w^4}\right),\\
K_2 &= \frac{1}{m_W^2}\left[\frac{9+g5s}{6m_W^2}\left(\sum_l m^4_{e_l}+N_c\sum_{l,m}V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m^4_{U_l}+m^4_{D_m}\right)\right)\right.\\
&\quad\left.+\frac{m_H^2}{2}\left(1+\frac{1}{2c_w^2}\right)\right]+\frac{1-12\lambda_{HV}}{4}\left(1+\frac{s_w^2}{3c_w^2}\left(1+\frac{1}{c_w^2}\right)\right),\\
K_3 &= \frac{1}{m_W^2}\left[\frac{9+g5s}{6m_W^2}\left(\sum_l m^4_{e_l}+N_c\sum_{i,j,k,l}\left(V_{U_i,D_j}V^\dagger_{D_j,U_k}V_{U_k,D_l}V^\dagger_{D_l,U_i}\right.\right.\right.\\
&\quad\left.\left.\left(m^2_{U_i}m^2_{U_k}+m^2_{D_j}m^2_{D_l}+\frac{1-g5s}{4}m_{U_i}m_{U_k}m_{D_j}m_{D_l}\right)\right)\right)\\
&\quad\left.+\frac{m_H^2}{2}\left(1+\frac{1}{2c_w^2}\right)\right]+(1-12\lambda_{HV})\left(\frac{1+s_w^4}{4}+\frac{1}{6}\left(s_w^2+\frac{2s_w^6}{c_w^2}\right)+\frac{s_w^8}{12c_w^4}\right)
\end{aligned}
\tag{4.42}
$$

Vector-Vector-Vector-Vector vertices

The generic effective vertex is shown in Fig.6 (b) with the following expression

$$
\begin{aligned}
\mathrm{Vert}\left(V_{1\mu},V_{2\nu},V_{3\rho},V_{4\sigma}\right) = \frac{ie^4}{\pi^2}\;&\left(C_1^{V_1V_2V_3V_4}g_{\mu\nu}g_{\rho\sigma}+C_2^{V_1V_2V_3V_4}g_{\mu\rho}g_{\nu\sigma}\right.\\
&\left.+C_3^{V_1V_2V_3V_4}g_{\mu\sigma}g_{\nu\rho}+C_0^{V_1V_2V_3V_4}\varepsilon_{\mu\nu\rho\sigma}\right),
\end{aligned}
\tag{4.43}
$$

with the actual values of V_1, V_2, V_3, V_4, $C_1^{V_1V_2V_3V_4}$,$C_2^{V_1V_2V_3V_4}$, $C_3^{V_1V_2V_3V_4}$ and $C_0^{V_1V_2V_3V_4}$

$$
\begin{aligned}
C_1^{AAAA} &= \frac{1}{12}\left(-1+\sum_l Q_{L_l}^4 + N_c\sum_l Q_{Q_l}^4\right),\\
C_2^{AAAA} &= C_3^{AAAA} = C_1^{AAAA},\\
C_0^{AAAA} &= 0,\\
C_1^{AAAZ} &= \frac{1}{12}\left[\frac{c_w}{s_w}+\sum_l\left(\frac{s_w}{c_w}Q_{L_l}^4-\frac{1}{2s_wc_w}Q_{L_l}^3I_{3L_l}\right)\right.\\
&\qquad \left.+N_c\sum_l\left(\frac{s_w}{c_w}Q_{Q_l}^4-\frac{1}{2s_wc_w}Q_{Q_l}^3I_{3Q_l}\right)\right],\\
C_2^{AAAZ} &= C_3^{AAAZ} = C_1^{AAAZ},\\
C_0^{AAAZ} &= 0,\\
C_1^{AAZZ} &= \frac{1}{12}\left[-\frac{c_w^2}{s_w^2}+\frac{1}{2}\sum_l\left(\frac{s_w^2}{c_w^2}Q_{L_l}^4+\left(\frac{s_w}{c_w}Q_{L_l}^2-\frac{1}{s_wc_w}Q_{L_l}I_{3L_l}\right)^2\right)\right.\\
&\qquad +\frac{N_c}{2}\sum_l\left(\frac{s_w^2}{c_w^2}Q_{Q_l}^4+\left(\frac{s_w}{c_w}Q_{Q_l}^2-\frac{1}{s_wc_w}Q_{Q_l}I_{3Q_l}\right)^2\right)\\
&\qquad \left.+\frac{1-g5s}{2}\ 2\left(\sum_l\frac{1}{s_w^2c_w^2}Q_{L_l}^2I_{3L_l}^2+N_c\sum_l\frac{1}{s_w^2c_w^2}Q_{Q_l}^2I_{3Q_l}^2\right)\right],\\
C_2^{AAZZ} &= \frac{1}{12}\left[-\frac{c_w^2}{s_w^2}+\frac{1}{2}\sum_l\left(\frac{s_w^2}{c_w^2}Q_{L_l}^4+\left(\frac{s_w}{c_w}Q_{L_l}^2-\frac{1}{s_wc_w}Q_{L_l}I_{3L_l}\right)^2\right)\right.\\
&\qquad \left.+\frac{N_c}{2}\sum_l\left(\frac{s_w^2}{c_w^2}Q_{Q_l}^4+\left(\frac{s_w}{c_w}Q_{Q_l}^2-\frac{1}{s_wc_w}Q_{Q_l}I_{3Q_l}\right)^2\right)\right],\\
C_3^{AAZZ} &= C_2^{AAZZ},\\
C_0^{AAZZ} &= 0,\\
C_1^{AZZZ} &= \frac{1}{12}\left[\frac{c_w^3}{s_w^3}+\sum_l\left(\frac{s_w^3}{c_w^3}Q_{L_l}^4-\frac{3s_w}{2c_w}Q_{L_l}^3I_{3L_l}+\frac{5-2g5s}{2s_wc_w^3}Q_{L_l}^2I_{3L_l}^2\right.\right.\\
&\qquad \left.-\frac{2-g5s}{2s_w^3c_w^3}Q_{L_l}I_{3L_l}^3\right)+N_c\sum_l\left(\frac{s_w^3}{c_w^3}Q_{Q_l}^4-\frac{3s_w}{2c_w^3}Q_{Q_L}^3I_{3Q_l}\right.\\
&\qquad \left.\left.+\frac{5-2g5s}{2s_wc_w^3}Q_{Q_l}^2I_{3Q_l}^2-\frac{2-g5s}{2s_w^3c_w^3}Q_{Q_l}I_{Q_l}^3\right)\right],\\
C_2^{AZZZ} &= C_3^{AZZZ} = C_1^{AZZZ},\\
C_0^{AZZZ} &= 0,
\end{aligned}
$$

$$
\begin{aligned}
C_1^{ZZZZ} &= \frac{1}{12}\Bigg[-\frac{c_w^4}{s_w^4}+\sum_l\Bigg(\frac{s_w^4}{c_w^4}Q_{L_l}^4-\frac{2s_w^2}{c_w^2}Q_{L_l}^3I_{3L_l}+\frac{5-2g5s}{c_w^4}Q_{L_l}^2I_{3L_l}^2\\
&\quad-\frac{4-2g5s}{s_w^2c_w^4}Q_{L_l}I_{3L_l}^3+\frac{5-3g5s}{4s_w^4c_w^4}I_{3L_l}^4\Bigg)\\
&\quad+N_c\sum_l\Bigg(\frac{s_w^4}{c_w^4}Q_{Q_l}^4-\frac{2s_w^2}{c_w^4}Q_{Q_l}^3I_{3Q_l}+\frac{5-2g5s}{c_w^4}Q_{Q_l}^2I_{3Q_l}^2\\
&\quad-\frac{4-2g5s}{s_w^2c_w^4}Q_{Q_l}I_{3Q_l}^3+\frac{5-3g5s}{4s_w^4c_w^4}I_{3Q_l}^4\Bigg)\Bigg],\\
C_2^{ZZZZ} &= C_3^{ZZZZ},\\
C_0^{ZZZZ} &= 0,\\
C_1^{AAW^-W^+} &= \frac{1}{16s_w^2}\left[\frac{10+4\lambda_{HV}}{3}+3\ \frac{7-g5s}{6}+\frac{(51-g5s)\,N_c}{54}\sum_{l,m}V_{U_l,D_m}V_{D_m,U_l}^\dagger\right],\\
C_2^{AAW^-W^+} &= -\frac{1}{16s_w^2}\left[\frac{7+2\lambda_{HV}}{3}+1+\frac{11N_c}{27}\sum_{l,m}V_{U_l,D_m}V_{D_m,U_l}^\dagger\right],\\
C_3^{AAW^-W^+} &= C_2^{AAW^-W^+},\\
C_0^{AAW^-W^+} &= \frac{1+g5s}{2}\ \frac{i}{48s_w^2}\left(N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\right)-9\right),\\
C_1^{AZW^-W^+} &= \frac{1}{16s_wc_w}\Bigg[-\frac{(10+4\lambda_{HV})\,c_w^2}{3s_w^2}+3\left(\frac{7-g5s}{6}-\frac{10+g5s}{12s_w^2}\right)\\
&\quad+N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\left(\frac{51-g5s}{54}-\frac{10+g5s}{12s_w^2}\right)V_{U_l,D_m}V_{D_m,U_l}^\dagger\right)\Bigg],\\
C_2^{AZW^-W^+} &= \frac{1}{16s_wc_w}\Bigg[\frac{(7+2\lambda_{HV})\,c_w^2}{3s_w^2}+3\left(\frac{4+g5s}{12s_w^2}-\frac{1}{3}\right)\\
&\quad+N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\left(\frac{4+g5s}{12s_w^2}-\frac{11}{27}\right)\right)\Bigg],\\
C_3^{AZW^-W^+} &= C_2^{AZW^-W^+},\\
C_0^{AZW^-W^+} &= \frac{i}{192c_ws_w^3}\left(\frac{1+g5s}{2}\ \left(4s_w^2-3\right)+\frac{1-g5s}{2}\ 2\right)\left(N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\right)-9\right),\\
C_1^{ZZW^-W^+} &= \frac{(5+2\lambda_{HV})\,c_w^2}{24s_w^4}+\frac{1}{16c_w^2}\Bigg[3\left(\frac{7-g5s}{6}-\frac{10+g5s}{6s_w^2}+\frac{23-g5s}{24s_w^4}\right)\\
&\quad+N_c\left(\frac{51-g5s}{54}-\frac{10+g5s}{6s_w^2}+\frac{23-g5s}{24s_w^4}\right)\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\right)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
C_2^{ZZW^-W^+} &= -\frac{(7+2\lambda_{HV})\,c_w^2}{48 s_w^4} + \frac{1}{16 c_w^2}\left[3\left(-\frac{1}{3} + \frac{4+g5s}{6s_w^2} - \frac{9+g5s}{24 s_w^4}\right)\right.\\
&\quad \left. + N_c\left(-\frac{11}{27} + \frac{4+g5s}{6s_w^2} - \frac{9+g5s}{24 s_w^4}\right)\sum_{l,m} V_{U_l,D_m} V^\dagger_{D_m,U_l}\right],\\
C_3^{ZZW^-W^+} &= C_2^{ZZW^-W^+},\\
C_0^{ZZW^-W^+} &= -\frac{i\,(12\,(1+g5s) + (g5s-1)\,c_w^2)}{1152 s_w^2 c_w^2}\left(N_c \sum_{l,m}\left(V_{U_l,D_m} V^\dagger_{D_m,U_l}\right) - 9\right),\\
C_1^{W^-W^+W^-W^+} &= \frac{1}{16 s_w^4}\left[\frac{3+2\lambda_{HV}}{3} + 3\frac{7-g5s}{12}\right.\\
&\quad \left. + \frac{(7-g5s)\,N_c}{12}\sum_{i,j,k,m}\left(V_{U_i,D_j} V^\dagger_{D_j,U_k} V_{U_k,D_m} V^\dagger_{D_m,U_i}\right)\right],\\
C_2^{W^-W^+W^-W^+} &= -\frac{1}{8 s_w^4}\left[\frac{7+2\lambda_{HV}}{3} + 3\frac{9+g5s}{24}\right.\\
&\quad \left. + \frac{(9+g5s)\,N_c}{24}\sum_{i,j,k,m}\left(V_{U_i,D_j} V^\dagger_{D_j,U_k} V_{U_k,D_m} V^\dagger_{D_m,U_i}\right)\right],\\
C_3^{W^-W^+W^-W^+} &= C_1^{W^-W^+W^-W^+},\\
C_0^{W^-W^+W^-W^+} &= 0.
\end{aligned}
\tag{4.44}
$$

Scalar-Scalar-Vector-Vector vertices

The generic effective vertex is shown in Fig.6 (c) with the following expression

$$
\mathrm{Vert}\,(S_1, S_2, V_{1\mu}, V_{2\nu}) = \frac{ie^4}{\pi^2}\; C^{S_1 S_2 V_1 V_2}\; g_{\mu\nu}, \tag{4.45}
$$

with the actual values of S_1, S_2, V_1, V_2 and $C^{S_1 S_2 V_1 V_2}$

$$
\begin{aligned}
C^{H\phi^0 AA} &= C^{H\phi^0 AZ} = C^{H\phi^0 ZZ} = C^{H\phi^0 W^+W^-} = 0, \\
C^{HHAA} &= \frac{1}{16 s_w^2}\left[\frac{1}{12} - \frac{1}{m_W^2}\left(\sum_l \left(Q_{L_l}^2 m_{L_l}^2\right) + N_c \sum_l \left(Q_{Q_l}^2 m_{Q_l}^2\right)\right)\right], \\
C^{\phi^0\phi^0 AA} &= \frac{1}{16 s_w^2}\left[\frac{1}{12} - \frac{7-4\ g5s}{3 m_W^2}\left(\sum_l \left(Q_{L_l}^2 m_{L_l}^2\right) + N_c \sum_l \left(Q_{Q_l}^2 m_{Q_l}^2\right)\right)\right], \\
C^{HHAZ} &= \frac{1}{16 s_w}\left[\frac{4+s_w^2}{12 s_w^2 c_w} + \frac{1}{m_W^2 c_w}\left(\sum_l \left(Q_{L_l} m_{L_l}^2 \left(\frac{I_{3L_l}}{2 s_w^2} - Q_{L_l}\right)\right)\right.\right. \\
&\left.\left. + N_c \sum_l \left(Q_{Q_l} m_{Q_l}^2 \left(\frac{I_{3Q_l}}{2 s_w^2} - Q_{Q_l}\right)\right)\right)\right], \\
C^{\phi^0\phi^0 AZ} &= \frac{1}{16 s_w}\left[\frac{4+s_w^2}{12 s_w^2 c_w} + \frac{7-4g5s}{3 m_W^2 c_w}\left(\sum_l \left(Q_{L_l} m_{L_l}^2 \left(\frac{I_{3L_l}}{2 s_w^2} - Q_{L_l}\right)\right)\right.\right. \\
&\left.\left. + N_c \sum_l \left(Q_{Q_l} m_{Q_l}^2 \left(\frac{I_{3Q_l}}{2 s_w^2} - Q_{Q_l}\right)\right)\right)\right], \\
C^{HHZZ} &= -\frac{1}{16 c_w^2}\left[\frac{1+2c_w^2+40c_w^4-4c_w^6}{48 s_w^4 c_w^2}\right. \\
&+ \frac{1}{m_W^2}\left(\sum_l \left(m_{L_l}^2\left(Q_{L_l}^2 + \frac{(7+g5s)\, I_{3L_l}^2}{6 s_w^4} - \frac{Q_{L_l} I_{3L_l}}{s_w^2}\right)\right)\right. \\
&\left.\left. + N_c \sum_l \left(m_{Q_l}^2\left(Q_{Q_l}^2 + \frac{(7+g5s)\, I_{3Q_l}^2}{6 s_w^4} - \frac{Q_{Q_l} I_{3Q_l}}{s_w^2}\right)\right)\right)\right], \\
C^{\phi^0\phi^0 ZZ} &= -\frac{1}{16 c_w^2}\left[\frac{1+2c_w^2+40c_w^4-4c_w^6}{48 s_w^4 c_w^2}\right. \\
&+ \frac{7-4g5s}{3 m_W^2}\left(\sum_l \left(m_{L_l}^2\left(Q_{L_l}^2 + \frac{(71+17\ g5s)\, I_{3L_l}^2}{66 s_w^4} - \frac{Q_{L_l} I_{3L_l}}{s_w^2}\right)\right)\right. \\
&\left.\left. + N_c \sum_l \left(m_{Q_l}^2\left(Q_{Q_l}^2 + \frac{(71+17\ g5s)\, I_{3Q_l}^2}{66 s_w^4} - \frac{Q_{Q_l} I_{3Q_l}}{s_w^2}\right)\right)\right)\right], \\
C^{HHW^-W^+} &= -\frac{1}{48 s_w^4}\left[\frac{1+38c_w^2}{16 c_w^2}\right. \\
&\left. + \frac{7+g5s}{8 m_W^2}\left(\sum_l m_{e_l}^2 + N_c \sum_{l,m}\left(V_{U_l,D_m} V_{D_m,U_l}^\dagger \left(m_{U_l}^2 + m_{D_m}^2\right)\right)\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
C^{\phi^0\phi^0W^-W^+} &= -\frac{1}{48s_w^4}\left[\frac{1+38c_w^2}{16c_w^2}+\frac{11-3\ g5s}{8m_W^2}\left(\sum_l m_{e_l}^2\right.\right.\\
&\qquad \left.\left.+N_c\sum_{l,m}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}^2+m_{D_m}^2-\frac{1-g5s}{2}\ \frac{4}{7}m_{U_l}m_{D_m}\right)\right)\right)\right],\\
C^{H\phi^+AW^-} &= C^{H\phi^-AW^+}=\frac{1}{24s_w^3}\left(\frac{1+22c_w^2}{32c_w^2}+K_1\right),\\
C^{\phi^0\phi^+AW^-} &= -\frac{i}{24s_w^3}\left(\frac{1+22c_w^2}{32c_w^2}+K_2\right),\\
C^{\phi^0\phi^-AW^+} &= -C^{\phi^0\phi^+AW^-},\\
C^{H\phi^+ZW^-} &= C^{H\phi^-ZW^+}=\frac{s_w}{c_w}\ C^{H\phi^+AW^-},\\
C^{\phi^0\phi^+ZW^-} &= -\frac{i}{24s_w^2c_w}\left(\frac{1+22c_w^2}{32c_w^2}+K_3\right),\\
C^{\phi^0\phi^-ZW^+} &= -C^{\phi^0\phi^+ZW^-},\\
C^{\phi^-\phi^+AA} &= -\frac{1}{12s_w^2}\left[\frac{1+21c_w^2}{16c_w^2}+\frac{1}{m_W^2}\left(\frac{9-g5s}{8}\sum_l m_{e_l}^2\right.\right.\\
&\qquad \left.\left.+\frac{5N_c}{6}\sum_{l,m}\left(V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}^2+m_{D_m}^2+\frac{1-g5s}{2}\ \frac{(m_{U_l}+m_{D_m})^2}{30}\right)\right)\right)\right],\\
C^{\phi^-\phi^+AZ} &= \frac{1}{12s_wc_w}\left\{\frac{42c_w^4-10c_w^2-1}{32s_w^2c_w^2}\right.\\
&\qquad -\frac{1}{m_W^2}\left[\sum_l\left(m_{e_l}^2Q_{e_l}\left(\frac{(9-g5s)\,Q_{e_l}}{8}+\frac{5I_{3\nu_l}}{8s_w^2}\right)\right)\right.\\
&\qquad +N_c\sum_{l,m}\left[V_{U_l,D_m}V^\dagger_{D_m,U_l}\left(m_{U_l}^2\left(\frac{5}{6}-\frac{I_{3D_m}}{s_w^2}\left(Q_{D_m}-\frac{5}{8}Q_{U_l}\right)\right)\right.\right.\\
&\qquad +m_{D_m}^2\left(\frac{5}{6}-\frac{I_{3U_l}}{s_w^2}\left(Q_{U_l}-\frac{5}{8}Q_{D_m}\right)\right)\\
&\qquad \left.\left.\left.\left.+\frac{1-g5s}{2}\ \frac{1}{36}\left(m_{U_l}+m_{D_m}\right)^2\right)\right]\right]\right\},
\end{aligned}
$$

$$
\begin{aligned}
C^{\phi^-\phi^+ZZ} &= \frac{1}{12c_w^2}\left\{\frac{-1+2c_w^2+44c_w^4-84c_w^6}{64s_w^4c_w^2} - \frac{1}{m_W^2}\left[\sum_l\left(m_{e_l}^2\left(\frac{(9-g5s)\,Q_{e_l}^2}{8}\right.\right.\right.\right. \\
&\quad \left.\left. + \frac{5Q_{e_l}I_{3\nu_l}}{4s_w^2} + \frac{I_{3\nu_l}^2}{s_w^4}\right)\right) + N_c\sum_{l,m}\left[V_{U_l,D_m}V_{D_m,U_l}^\dagger\right. \\
&\quad \left(m_{U_l}^2\left(\frac{5}{6} - \frac{I_{3D_m}}{s_w^2}\left(2Q_{D_m} - \frac{5}{4}Q_{U_l}\right) + \frac{I_{3D_m}^2}{s_w^4}\right)\right. \\
&\quad +m_{D_m}^2\left(\frac{5}{6} - \frac{I_{3U_l}}{s_w^2}\left(2Q_{U_l} - \frac{5}{4}Q_{D_m}\right) + \frac{I_{3U_l}^2}{s_w^4}\right) \\
&\quad \left.\left.\left.\left. + \frac{1-g5s}{2}\left(\frac{(m_{U_l}+m_{D_m})^2}{36} + \frac{m_{U_l}m_{D_m}}{8s_w^4}\right)\right)\right]\right]\right\}, \\
C^{\phi^-\phi^+W^-W^+} &= -\frac{1}{48s_w^4}\left\{\frac{38c_w^2+1}{16c_w^2} + \frac{1}{m_W^2}\left[\left(\frac{9-g5s}{8}\sum_l m_{e_l}^2\right.\right.\right. \\
&\quad + \frac{N_c}{4}\sum_{i,j,k,l}\left(V_{U_i,D_j}V_{D_j,U_k}^\dagger V_{U_k,D_l}V_{D_l,U_i}^\dagger\left(2m_{U_l}^2 + 2m_{U_k}^2 + 2m_{D_j}^2 + 2m_{D_l}^2\right.\right. \\
&\quad + \frac{1-g5s}{2}\left(m_{U_i}m_{U_k} + m_{U_i}m_{D_j} + m_{U_i}m_{D_l} + m_{U_k}m_{D_j}\right. \\
&\quad \left.\left.\left.\left.\left.\left. + m_{U_k}m_{D_l} + m_{D_j}m_{D_l}\right)\right)\right)\right)\right]\right\}. \qquad (4.46)
\end{aligned}
$$

where

$$
\begin{aligned}
K_1 &= \frac{1}{8m_W^2}\left[\sum_l m_{e_l}^2 + N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\left(3m_{D_m}^2 + 2m_{U_l}^2\right)\right)\right], \\
K_2 &= \frac{1}{8m_W^2}\left[(-1+2g5s)\sum_l m_{e_l}^2\right. \\
&\quad \left. + N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\left(\frac{7+2g5s}{3}m_{D_m}^2 + \frac{2+4g5s}{3}m_{U_l}^2\right)\right)\right], \\
K_3 &= \frac{1}{8m_W^2}\left[(-1+2g5s)\sum_l m_{e_l}^2 + \frac{1-g5s}{2}\,\frac{1}{s_w^2}\sum_l m_{e_l}^2\right. \\
&\quad + N_c\sum_{l,m}\left(V_{U_l,D_m}V_{D_m,U_l}^\dagger\left(\frac{7+2g5s}{3}m_{D_m}^2 + \frac{2+4g5s}{3}m_{U_l}^2\right.\right. \\
&\quad \left.\left.\left. + \frac{1-g5s}{2}\,\frac{m_{U_l}^2 + m_{D_m}^2}{s_w^2}\right)\right)\right]. \qquad (4.47)
\end{aligned}
$$

All possible non-vanishing R effective vertices in SM are listed above. Our results are the same as those in Refs.[17, 18] in the same γ_5 scheme. In addition, all terms

proportional to ε evidently vanish in the formulae after including all fermions in SM. This is guaranteed by the cancelation of gauge anomaly in SM.

5. Summary

In summary, we have studied Feynman rules for the rational part R of the Standard Model amplitudes at one-loop level in the 't Hooft-Veltman γ_5 scheme. Comparing Feynman rules obtained in this scheme for quantum chromodynamics and electroweak 1-loop amplitudes with that obtained in another γ_5 scheme (the KKS scheme) in Refs.[17, 18], we find the latter result can be recovered after setting $g5s = 1$ in our results. As an independent check, we also calculated Feynman rules in the KKS scheme, and found results completely in agreement with the analytical expressions given in their updated version, as mentioned in Ref.[33].

With these rational terms, one can simplify fermion chains or Dirac traces in 4 dimensions at the amplitude level, resulting in more convenient general calculations for multi-leg processes, compared with directly solving d dimension Dirac algebras. To clarify some existing ambiguities in DREG, we give our results in FDH and HV DREG, as well as in KKS and HV γ_5 schemes. To be more reliable, our results are checked and found to be in agreement with previously published works (see Refs.[17, 18] and Ref.[33]) in the KKS γ_5 scheme. In particular, the expressions for R in the HV γ_5 scheme are given in this paper to meet the needs of some specific NLO calculations.

As the HV γ_5 scheme is the only rigorously proved consistent regime to all orders, our results in this γ_5 scheme are surely useful and necessary in clarifying the γ_5 problem in DREG. The only dimensional regularization dependent terms (except scaleless bubbles) in one-loop calculations are rational terms R. These Feynman rules are also helpful in eliminating or finding the uncertainties arising from freedoms in DREG. For instance, in NLO QCD corrections to the W boson hadronic decays, the rational terms R in HV and KKS γ_5 schemes are different (see Eq.(4.6)), being right-handed in HV scheme while left-handed in KKS scheme. These differences are the only differences after including virtual parts, counter terms, and real radiations in these two γ_5 schemes. Actually, the violation of the anti-commutation relation between γ_5 and γ_μ in HV spoils the Ward identity, and a finite renormalization is needed in this scheme to obtain the same expressions in HV and KKS (see the details in Ref.[32]).

Acknowledgments

We would like to thank R. Pittau for kindly checking some of our results and helpful communications. We also thank Y.Q. Ma, K. Wang and D. Li for the assistance in

writing this paper. This work was supported by the National Natural Science Foundation of China (No.10805002, No.11021092, No.11075002, No.11075011), the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant No. 201020), and the Ministry of Science and Technology of China (2009CB825200).

References

[1] A. Bredenstein, A. Denner, S. Dittmaier, and S. Pozzorini, *NLO QCD corrections to top anti-top bottom anti-bottom production at the LHC: 1. quark-antiquark annihilation*, *JHEP* **08** (2008) 108, [arXiv:0807.1248];

A. Bredenstein, A. Denner, S. Dittmaier, and S. Pozzorini, *NLO QCD corrections to top anti-top bottom anti-bottom production at the LHC: 2. full hadronic results*, *JHEP* **03** (2010) 021, [arXiv:1001.4006].

[2] G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau, and M. Worek, *Assault on the NLO Wishlist: $pp \to t\bar{t}b\bar{b}$*, *JHEP* **09** (2009) 109, [arXiv:0907.4723].

[3] R. K. Ellis, K. Melnikov, and G. Zanderighi, *W+3 jet production at the Tevatron*, *Phys. Rev.* **D80** (2009) 094002, [arXiv:0906.1445];

C. F. Berger *et. al.*, *Next-to-Leading Order QCD Predictions for W+3-Jet Distributions at Hadron Colliders*, *Phys. Rev.* **D80** (2009) 074036, [arXiv:0907.1984];

C. F. Berger *et. al.*, *Next-to-Leading Order QCD Predictions for $Z, gamma^* + 3 - Jet$ Distributions at the Tevatron*, *Phys. Rev.* **D82** (2010) 074002, [arXiv:1004.1659].

[4] C. F. Berger *et. al.*, *Precise Predictions for W + 4 Jet Production at the Large Hadron Collider*, *Phys. Rev. Lett.* **106** (2011) 092001, [arXiv:1009.2338].

[5] T. Melia, K. Melnikov, R. Rontsch, and G. Zanderighi, *Next-to-leading order QCD predictions for W+W+jj production at the LHC*, *JHEP* **12** (2010) 053, [arXiv:1007.5313].

G. Bozzi, B. Jager, C. Oleari, and D. Zeppenfeld, *Next-to-leading order QCD corrections to W+Z and W-Z production via vector-boson fusion*, *Phys. Rev.* **D75** (2007) 073004, [hep-ph/0701105];

B. Jager, C. Oleari, and D. Zeppenfeld, *Next-to-leading order QCD corrections to Z boson pair production via vector-boson fusion*, *Phys. Rev.* **D73** (2006) 113006, [hep-ph/0604200];

B. Jager, C. Oleari, and D. Zeppenfeld, *Next-to-leading order QCD corrections to W+ W- production via vector-boson fusion*, *JHEP* **07** (2006) 015, [hep-ph/0603177].

[6] F. Campanario, *Towards $pp \to VVjj$ at NLO QCD: Bosonic contributions to triple vector boson production plus jet*, arXiv:1105.0920.

[7] G. Bevilacqua, M. Czakon, C. G. Papadopoulos, and M. Worek, *Dominant QCD Backgrounds in Higgs Boson Analyses at the LHC: A Study of $pp \to t\bar{t} + 2$ jets at Next-To-Leading Order*, *Phys. Rev. Lett.* **104** (2010) 162002, [arXiv:1002.4009];

G. Bevilacqua *et. al.*, *NLO QCD calculations with HELAC-NLO*, *Nucl. Phys. Proc. Suppl.* **205-206** (2010) 211–217, [arXiv:1007.4918];

M. Worek, *Recent developments in NLO QCD calculations: the particular case of $pp \to t\bar{t}\, jj$*, arXiv:1012.4987.

[8] A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, *NLO QCD corrections to $WWb\bar{b}$ production at hadron colliders*, *Phys. Rev. Lett* **106** (2011) 052001, [arXiv:1012.3975];

G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos and M. Worek, *Complete off-shell effects in top quark pair hadroproduction with leptonic decay at next-to-leading order*, *JHEP* **1102** (2011) 083, [arXiv:1012.4230].

[9] T. Binoth *et. al.*, *Next-to-leading order QCD corrections to $pp \to b\bar{b}b\bar{b} + X$ at the LHC: the quark induced case*, *Phys. Lett.* **B685** (2010) 293–296, [arXiv:0910.4379];

N. Greiner, A. Guffanti, T. Reiter and J. Reuter, *NLO QCD corrections to the production of two bottom-antibottom pairs at the LHC*,arXiv:1105.3624.

[10] F. Campanario, C. Englert, M. Rauch and D. Zeppenfeld, *Precise predictions for W $\gamma\ \gamma$ +jet production at hadron colliders*, [arXiv:1106.4009].

[11] **SM and NLO Multileg Working Group** Collaboration, J. R. Andersen *et. al.*, *The SM and NLO multileg working group: Summary report*, arXiv:1003.1241.

[12] Z. Bern, L. J. Dixon, and D. A. Kosower, *One loop corrections to five gluon amplitudes*, *Phys. Rev. Lett.* **70** (1993) 2677–2680, [hep-ph/9302280];

Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, *One-Loop n-Point Gauge Theory Amplitudes, Unitarity and Collinear Limits*, *Nucl. Phys.* **B425** (1994) 217–260, [hep-ph/9403226];

Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, *Fusing gauge theory tree amplitudes into loop amplitudes*, *Nucl. Phys.* **B435** (1995) 59–101, [hep-ph/9409265];

Z. Bern, L. J. Dixon, and D. A. Kosower, *One loop corrections to two quark three gluon amplitudes*, *Nucl. Phys.* **B437** (1995) 259–304, [hep-ph/9409393];

R. Britto, F. Cachazo, and B. Feng, *Generalized unitarity and one-loop amplitudes in $N = 4$ super-Yang-Mills*, *Nucl. Phys.* **B725** (2005) 275–305, [hep-th/0412103].

[13] F. del Aguila and R. Pittau, *Recursive numerical calculus of one-loop tensor integrals*, *JHEP* **07** (2004) 017, [hep-ph/0404120];

R. Pittau, *Formulae for a numerical computation of one-loop tensor integrals*, hep-ph/0406105;

G. Ossola, C. G. Papadopoulos, and R. Pittau, *Reducing full one-loop amplitudes to scalar integrals at the integrand level*, *Nucl. Phys.* **B763** (2007) 147–169, [hep-ph/0609007];

G. Ossola, C. G. Papadopoulos, and R. Pittau, *Numerical Evaluation of Six-Photon Amplitudes*, *JHEP* **07** (2007) 085, [arXiv:0704.1271];

G. Ossola, C. G. Papadopoulos, and R. Pittau, *CutTools: a program implementing the OPP reduction method to compute one-loop amplitudes*, *JHEP* **03** (2008) 042, [arXiv:0711.3596];

P. Mastrolia, G. Ossola, C. G. Papadopoulos, and R. Pittau, *Optimizing the Reduction of One-Loop Amplitudes*, *JHEP* **06** (2008) 030, [arXiv:0803.3964].

[14] G. Ossola, C. G. Papadopoulos, and R. Pittau, *On the Rational Terms of the one-loop amplitudes*, *JHEP* **05** (2008) 004, [arXiv:0802.1876].

[15] T. Binoth, J. P. Guillet, and G. Heinrich, *Algebraic evaluation of rational polynomials in one-loop amplitudes*, *JHEP* **02** (2007) 013, [hep-ph/0609054].

[16] G. Passarino and M. J. G. Veltman, *One Loop Corrections for e^+e^- Annihilation Into $\mu^+\mu^-$ in the Weinberg Model*, *Nucl. Phys.* **B160** (1979) 151;

A. Denner and S. Dittmaier, *Reduction of one-loop tensor 5-point integrals*, *Nucl. Phys.* **B658** (2003) 175–202, [hep-ph/0212259];

A. Denner and S. Dittmaier, *Reduction schemes for one-loop tensor integrals*, *Nucl. Phys.* **B734** (2006) 62–115, [hep-ph/0509141].

[17] P. Draggiotis, M. V. Garzelli, C. G. Papadopoulos, and R. Pittau, *Feynman Rules for the Rational Part of the QCD 1-loop amplitudes*, *JHEP* **04** (2009) 072, [arXiv:0903.0356].

[18] M. V. Garzelli, I. Malamos, and R. Pittau, *Feynman rules for the rational part of the Electroweak 1- loop amplitudes*, *JHEP* **01** (2010) 040, [arXiv:0910.3130].

[19] M. V. Garzelli, I. Malamos, and R. Pittau, *Feynman rules for the rational part of the Electroweak 1- loop amplitudes in the R_{xi} gauge and in the Unitary gauge*, *JHEP* **1101** (2011) 029, [arXiv:1009.4302].

[20] M. V. Garzelli and I. Malamos, *R2SM: a package for the analytic computation of the R2 Rational terms in the Standard Model of the Electroweak interactions*, *Eur. Phys. J.* **C71** (2011) 1605,[arXiv:1010.1248].

[21] W. Siegel, *Supersymmetric Dimensional Regularization via Dimensional Reduction*, *Phys. Lett.* **B84** (1979) 193.

[22] Z. Bern and D. A. Kosower, *The Computation of loop amplitudes in gauge theories*, *Nucl. Phys.* **B379** (1992) 451–561;

Z. Kunszt, A. Signer, and Z. Trocsanyi, *One loop helicity amplitudes for all $2 \to 2$ processes in QCD and N=1 supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B411** (1994) 397–442, [hep-ph/9305239];

S. Catani, M. H. Seymour, and Z. Trocsanyi, *Regularization scheme independence and unitarity in QCD cross sections*, *Phys. Rev.* **D55** (1997) 6819–6829, [hep-ph/9610553];

Z. Bern, A. De Freitas, L. J. Dixon, and H. L. Wong, *Supersymmetric regularization, two-loop QCD amplitudes and coupling shifts*, *Phys. Rev.* **D66** (2002) 085002, [hep-ph/0202271].

[23] G. 't Hooft and M. J. G. Veltman, *Regularization and Renormalization of Gauge Fields*, *Nucl. Phys.* **B44** (1972) 189–213.

[24] D. Kreimer, *The Role of γ_5 in dimensional regularization*, hep-ph/9401354;

J. G. Korner, D. Kreimer, and K. Schilcher, *A Practicable γ_5 scheme in dimensional regularization*, *Z. Phys.* **C54** (1992) 503–512;

D. Kreimer, *The γ_5 problem and anomalies: A clifford algebra approach*, *Phys. Lett.* **B237** (1990) 59.

[25] C. G. Bollini and J. J. Giambiagi, *Lowest order divergent graphs in nu-dimensional space*, *Phys. Lett.* **B40** (1972) 566–568;

G. M. Cicuta and E. Montaldi, *Analytic renormalization via continuous space dimension*, *Nuovo Cim. Lett.* **4** (1972) 329–332;

J. F. Ashmore, *A Method of Gauge Invariant Regularization*, *Lett. Nuovo Cim.* **4** (1972) 289–290.

[26] P. Breitenlohner and D. Maison, *Dimensional Renormalization and the Action Principle*, *Commun. Math. Phys.* **52** (1977) 11-38;

P. Breitenlohner and D. Maison, *Dimensionally Renormalized Green's Functions for Theories with Massless Particles. 1*, *Commun. Math. Phys.* **52** (1977) 39;

P. Breitenlohner and D. Maison, *Dimensionally Renormalized Green's Functions for Theories with Massless Particles. 2*, *Commun. Math. Phys.* **52** (1977) 55.

G. Bonneau, *Consistency in dimensional regularization with γ_5*, *Phys. Lett.* **B96** (1980) 147;

G. Bonneau, *Preserving canonical ward identities in dimensional regularization with a nonanticommuting γ_5*, *Nucl. Phys.* **B177** (1981) 523.

[27] C. P. Martin and D. Sanchez-Ruiz, *Action principles, restoration of BRS symmetry and the renormalization group equation for chiral nonAbelian gauge theories in dimensional renormalization with a nonanticommuting gamma(5)*, *Nucl. Phys.* **B572** (2000) 387, [hep-th/9905076].

[28] C. Schubert, *THE YUKAWA MODEL AS AN EXAMPLE FOR DIMENSIONAL RENORMALIZATION WITH gamma(5)*, *Nucl. Phys.* **B323** (1989) 478;

M. Pernici, M. Raciti and F. Riva, *Dimensional renormalization of Yukawa theories via Wilsonian methods*, *Nucl. Phys.* **B577** (2000) 293, [hep-th/9912248];

M. Pernici, *Seminaive dimensional renormalization*, *Nucl. Phys.* **B582** (2000) 733, [hep-th/9912278];

M. Pernici and M. Raciti, *Axial current in QED and seminaive dimensional renormalization*, *Phys. Lett.* **B513** (2001) 421, [hep-th/0003062];

R. Ferrari, A. Le Yaouanc, L. Oliver and J. C. Raynal, *Gauge invariance and dimensional regularization with gamma(5) in flavor changing neutral processes*, *Phys. Rev.* **D52** (1995) 3036;

F. Jegerlehner, *Facts of life with gamma(5)*, *Eur. Phys. J.* **C18** (2001) 673, [hep-th/0005255].

[29] J. Kublbeck, M. Bohm, and A. Denner, *FEYN ARTS: COMPUTER ALGEBRAIC GENERATION OF FEYNMAN GRAPHS AND AMPLITUDES*, *Comput. Phys. Commun.* **60** (1990) 165–180;

T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3*, *Comput. Phys. Commun.* **140** (2001) 418–431, [hep-ph/0012260].

[30] A. Denner, *Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200*, *Fortschr. Phys.* **41** (1993) 307–420, [arXiv:0709.1075].

[31] J. G. Korner, N. Nasrallah, and K. Schilcher, *Evaluation of the flavor changing vertex $b \to sh$ using the breitenlohner-maison-'t hooft-veltman γ_5 scheme*, *Phys. Rev.* **D41** (1990) 888.

[32] H. S. Shao, Y. J. Zhang and K. T. Chao, *Dijet Invariant Mass Distribution in Top Quark Hadronic Decay with QCD Corrections*, arXiv:1106.5483.

[33] M. V. Garzelli, I. Malamos, and R. Pittau, *Erratum: Feynman rules for the rational part of the Electroweak 1-loop amplitudes* , *JHEP* **10** (2010) 097.